

Seat No. \_\_\_\_\_

Enrollment No. \_\_\_\_\_

**GUJARAT TECHNOLOGICAL UNIVERSITY**

**P.D.D.C Sem – I Examination, January – 2010**

**Subject code: X10001**

**Subject Name : Mathematics - I**

**Date: 01 / 01 / 2010**

**Time: 11.00 am – 2.00 pm**

**Total marks:70**

**Instructions:**

**1. Attempt all questions.**

**2. Make suitable assumption wherever necessary.**

**3. Figure to the right indicate full marks.**

**Q.1.** Do as directed.

(a) Find vector normal to the surface  $x^2 + y^2 - z = 1$  at the point (1, 1, 1). (02)

(b) Solve the differential equation  $\frac{dy}{dx} + \frac{1}{x}y = x^2y^6$ . (03)

(c) Trace the curve  $y^2(a-x) = x^2(a+x)$ . (03)

(d) Determine Rank of the following matrix by row echelon form (03)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 7 & 8 & 9 & 10 \\ 12 & 13 & 14 & 15 \end{bmatrix}$$

(e) Using Gauss-Jordan method, find inverse of following Matrix (03)

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

**Q.2.** Attempt the following:

(a) Solve the following system of equations: (03)

$$x + y + z = 6;$$

$$x + 2y + 3z = 14; \text{ by Gaussian elimination and back substitution.}$$

$$x + 4y + 9z = 36.$$

(b) Find Eigen values and Eigen vectors of following Matrix (04)

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

(c) Solve the following differential equation

(i)  $x^4 \frac{dy}{dx} + x^3y - \sec xy = 0$  (04)

(ii)  $(5x^4 + 6x^2y^2 - 8xy^3)dx + (4x^3y - 12x^2y^2 - 5y^4)dy = 0$ . (03)

OR

(c) Solve the following differential equation

(i)  $x^2ydx - (x^3 + y^3)dy = 0$  (04)

(ii)  $(x+1) \frac{dy}{dx} - y = e^{3x}(x+1)^2$ . (03)

**Q.3.** Attempt the following:

(a) If  $V = r^m$ , where  $r^2 = x^2 + y^2 + z^2$  prove that (05)

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m(m+1)r^{m-2}.$$

(b) If  $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ , show that (05)

$$(i) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

$$(ii) x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u.$$

(c) If  $x = r \sin^2 \theta$ ,  $y = r \cos^2 \theta$ , prove that  $\frac{\partial(x, y)}{\partial(r, \theta)} = r \sin 2\theta$ . (04)

**OR**

**Q.3.** Attempt the following:

(a) If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$ , show that (05)

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x + y + z)^2}.$$

(b) If  $u = f\left(\frac{y}{x}\right) + x\phi\left(\frac{x}{y}\right) + 4xy$  prove that (05)

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 8xy.$$

(c) Find the approximate value of  $f(x, y) = \sqrt{x^2 + y^2}$  at the point (3.01, 4.02). (04)

**Q.4.** Attempt the following:

(a) Change the order of integration and evaluate (05)

$$\int_0^x \int_0^y x e^{-y} dy dx.$$

(b) Change into polar co-ordinate and evaluate (05)

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2) dy dx.$$

(c) Find the area bounded by the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$  (04)

**OR**

**Q.4.** Attempt the following:

(a) Evaluate  $\iint \frac{xy}{\sqrt{1-y^2}} dx dy$  over the +ve quadrant of the circle  $x^2 + y^2 = 1$ . (05)

(b) Evaluate  $\int_{-1}^1 \int_0^x \int_{x-y}^{x+y} (z - 2x - y) dz dy dx$ . (05)

- (c) Find the volume of the solid bounded by the surfaces  $x = 0, y = 0, z = 0$  and  $x + y + z = 1$ . (04)

**Q.5.** Attempt the following:

(a) If  $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2yz + 2z)\hat{k}$  (05)

Show that  $\vec{F}$  is both solenoidal and irrotational.

(b) If  $\phi = 2xy^2z + x^2y$ , evaluate  $\int_c \text{grad}\phi \cdot d\vec{r}$ , where  $c$  is the curve  $x = t$ , (05)

$y = t^2, z = t^3$  from  $t = 0$  to  $t = 1$ .

- (c) The charge  $Q$  on the plate of a condenser of capacity  $C$  charged through a resistance  $R$  by a steady voltage  $V$  satisfies the differential equation (04)

$$R \frac{dQ}{dt} + \frac{Q}{C} = V, \text{ if } Q = 0 \text{ at } t = 0, \text{ show that } i = \frac{V}{R} e^{-\frac{t}{RC}}.$$

**OR**

**Q.5.** Attempt the following:

(a) Find  $f(r)$  such that  $\nabla^2 f(r) = 0$ , where  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  (05)

(b) Verify Green's theorem for  $\oint_c [(xy + y^2)dx + x^2dy]$ , where  $c$  is (05)

bounded by  $y = x$  and  $y = x^2$ .

- (c) Find the orthogonal trajectories of confocal and coaxial parabolas (04)

$$r = \frac{2a}{1 + \cos\theta}.$$

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