

**GUJARAT TECHNOLOGICAL UNIVERSITY**

B.E. Sem-IV Remedial Examination Nov/ Dec. 2010

**Subject code: 140001****Subject Name: Mathematics-4****Date: 27 / 11/2010****Time: 03.00 pm – 06.00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a)** State the necessary and sufficient condition for a function to be analytic and prove the necessary condition. **05**

**(b)** Attempt the following. **09**

1. Show that if  $c$  is any  $n$ th root of unity other than unity itself, then  $1 + c + c^2 + \dots + c^{n-1} = 0$ .
2. Show that  $u(x, y) = 2x - x^3 + 3xy^2$  is harmonic in some domain and find a harmonic conjugate  $v(x, y)$ .
3. Discuss the convergence of  $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z-3i)^n$  and also find the radius of convergence.

**Q.2 (a)** Attempt the following.

1. Define bilinear transformation. Also find the bilinear transformation which maps the points  $z = 1, i, -1$  into the points  $w = i, 0, -i$ . Hence find the image of  $|z| < 1$ . **05**

2. Evaluate  $\int_C \bar{z} dz$ . Where  $C$  is the right-half of the circle  $|z| = 2$ . Hence show that  $\int_C \frac{dz}{z} = \pi i$ . **03**

**(b)** Attempt the following. **06**

1. Find the image of infinite strip  $\frac{1}{4} \leq y \leq \frac{1}{2}$  under the transformation  $w = \frac{1}{z}$ . Also show the region graphically.
2. Define residue at simple pole and find the sum of residues of the function  $f(z) = \frac{\sin z}{z \cos z}$  at its poles inside the circle  $|z| = 2$ .

**OR**

**(b)** Attempt the following. **06**

1. State Cauchy's integral formula and hence evaluate  $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ ; where  $C$  is the circle  $|z| = 3$ .

2. Expand  $f(z) = -\frac{1}{(z-1)(z-2)}$  in the region

- a)  $|z| < 1$    b)  $1 < |z| < 2$    c)  $2 < |z| < \infty$

**Q.3 (a)** State and prove Cauchy Goursat theorem. **05**

**(b)** Attempt the following. **09**

1. Write the two Laurent series expansion in powers of  $z$  that represent the function  $f(z) = \frac{1}{z^2(1-z)}$  in certain domains, and also specify domains.

2. Perform the five iterations of the bisection method to obtain a root of the equation  $f(x) = \cos x - xe^x = 0$ .

3. Compute the real root of  $f(x) = x - 2\sin x = 0$ , correct up to 6 decimal places using Secant method, starting from  $x_0 = 2$ ,  $x_1 = 1.9$ .

**OR**

**Q.3 (a)** Using an indentation along a Branch cut show that  $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$ . **05**

**(b)** Attempt the following. **09**

1. State Cauchy's residue theorem and evaluate  $\int_C \frac{5z-2}{z(z-1)} dz$ . Where  $C$  is the circle  $|z| = 2$ .

2. Use power method to find the largest of Eigen values of the matrix  $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ . Perform four iterations only.

3. Find a real root of the  $x^3 + x - 1 = 0$  correct up to six places of decimal places.

**Q.4 (a)** Compute  $\cosh(0.56)$  using Newton's forward difference formula and also estimate the error for the following table. **05**

$x$	0.5	0.6	0.7	0.8
$f(x)$	1.127626	1.185465	1.255169	1.337435

**(b)** Attempt the following. **09**

1. Solve the following linear system of equations by Gauss elimination method

$$8x_2 + 2x_3 = -7$$

$$3x_1 + 5x_2 + 2x_3 = 8$$

$$6x_1 + 2x_2 + 8x_3 = 26$$

2. State Trapezoidal rule with  $n=10$  and evaluate  $\int_0^1 e^{-x^2} dx$ .

3. Evaluate  $\int_1^3 \sin x dx$  using Gauss Quadrature of five points. Compare the result with analytic value.

OR

- Q.4 (a)** State Newton's divided difference interpolation formula and compute  $f(9.2)$  from the following data. **05**

$x_j$	8.0	9.0	9.5	11.0
$f(x)_j$	2.079442	2.197225	2.251292	2.397895

- (b)** Attempt the following. **09**

1. Solve the following linear system of equations by Gauss-Seidel  
 $10x_1 + x_2 + x_3 = 12$   
 $2x_1 + 10x_2 + x_3 = 13$   
 $2x_1 + 2x_2 + 10x_3 = 14$
2. The speed,  $v$  meters per second, of a car,  $t$  seconds after it starts, is show in the following table.

$t$	0	12	24	36	48	60	72	84	96	108	120
$v$	0	3.60	10.08	18.90	21.60	18.54	10.26	4.50	4.5	5.4	9.0

Using Simpson's  $\frac{1}{3}$  rule, find the distance travelled by the car in 2 minutes.

3. State Simpson's  $\frac{3}{8}$ -rule and evaluate  $\int_0^1 \frac{dx}{1+x^2}$  taking  $h = \frac{1}{6}$ .

- Q.5 (a)** Explain the Euler's method to find Numerical solution **05**  
of  $\frac{dy}{dx} = f(x, y)$ ,  $y(x_0) = y_0$ .

- (b)** Attempt the following. **09**

1. Use Runge-Kutta second order method to find the approximate value of  $y(0.2)$  given that  $\frac{dy}{dx} = x - y^2$  &  $y(0) = 1$  &  $h = 0.1$
2. Use Euler method to find  $y(1.4)$  given that  $\frac{dy}{dx} = xy^{\frac{1}{2}}$ ,  $y(1) = 1$
3. Use Taylor's series method to solve  $\frac{dy}{dx} = x^2y - 1$ ,  $y(0) = 1$ .  
Also find  $y(0.03)$ .

OR

- Q.5 (a)** Derive the Newton Raphson iterative scheme by drawing appropriate figure. **05**

- (b)** Attempt the following. **09**

1. Use Runge-Kutta fourth order method to find  $y(1.1)$ , given that  $\frac{dy}{dx} = x - y$ ,  $y(1) = 1$  and  $h = 0.05$ .

2. Find the Lagrange interpolating polynomial from the following data

$x$	0	1	4	5
$f(x)$	1	3	24	39

3. Set up a Newton iteration for computing the square root  $x$  of a given positive number  $c$  and apply it to  $c=2$

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