

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**BE- SEMESTER 1<sup>st</sup> / 2<sup>nd</sup> EXAMINATION (OLD SYLLABUS) – SUMMER - 2017**

**Subject Code: 110008****Date: 01/06/2017****Subject Name: Maths-I****Time: 2:30 PM to 05:30 PM****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Attempt the following
- (i) If  $5x \leq f(x) \leq 2x^2 + 2, \forall x \in R$  then find  $\lim_{x \rightarrow 2} f(x)$  **03**
- (ii) Find  $c$  by the mean value theorem for  $f(x) = \log x, x \in [1, e]$  **04**
- (b) Attempt the following
- (i) Evaluate  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$  **03**
- (ii) Expand  $e^x$  in powers of  $(x - 1)$  by Taylor's series. **04**
- Q.2** (a) Attempt the following
- (i) Trace the curve  $y^2(2a - x) = x^3$  **04**
- (ii) Check the convergence of  $\int_0^1 \frac{dx}{\sqrt{1-x}}$  **03**
- (b) Attempt the following
- (i) Check the convergence of  $\int_5^{\infty} \frac{7x+4}{x^2+9} dx$  **04**
- (ii) Find the extreme values for  $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$  **03**
- Q.3** (a) Attempt the following
- (i) Does the sequence  $\left\{ \frac{3}{n+3} \right\}$  monotone? **04**
- (ii) Test the convergence of  $\sum_{n=1}^{\infty} \frac{2n+1}{n^2+2n+1}$  **03**
- (b) Attempt the following
- (i) Test the convergence of  $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$  by Ratio Test **04**
- (ii) Prove that the series  $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$  is divergent. **03**
- Q.4** (a) If  $u = \tan^{-1} \left( \frac{x^3 + y^3}{x - y} \right)$ , show that  $xu_x + yu_y = \sin 2u$ . Also prove that  $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 2 \cos 3u \sin u$ . **07**
- (b) Attempt the following
- (i) If  $u = f(x - y, y - z, z - x)$ , prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$  **03**

- (ii) Find the equations of the tangent plane and normal line to the surface  $x^2 + 2y^2 + 3z^2 = 12$  at  $(1,2,-1)$ . **04**
- Q.5** (a) Attempt the following
- (i) Expand  $e^x \log(1+y)$  in powers of  $x$  and  $y$ . **04**
- (ii) Evaluate  $\int_{\pi/2}^{\pi} \int_1^2 x \cos xy \, dy \, dx$  **03**
- (b) Find a point on the plane  $2x + 3y - z = 5$  which is nearest to the origin. **07**
- Q.6** (a) Attempt the following
- (i) Evaluate  $\int_0^{\infty} \int_x^{\infty} e^{-y^2} \, dy \, dx$  by changing the order of integration. **04**
- (ii) Find the volume of the solid generated by revolving the region bounded by  $y^2 = x$  and the line  $x=l$ , about the line  $x=l$ . **03**
- (b) Attempt the following
- (i) Evaluate  $\int_0^{\log 2} \int_0^x \int_0^{\log y} e^{x+y+z} \, dz \, dy \, dx$  **04**
- (ii) Find the constants  $a, b, c$  so that  $\overline{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$  is irrotational. **03**
- Q.7** (a) Attempt the following
- (i) Find the area of the loop of the curve  $ay^2 = x^2(a-x)$ . **04**
- (ii) Determine  $\text{curl } \overline{F}$  at the point  $(2,0,3)$  given that  $\overline{F} = ze^{2xy}i + 2xy \cos yj + (x + 2y)k$ . **03**
- (b) Attempt the following
- (i) Using Green's theorem evaluate the integral  $\oint_C [(2x - y^2)dx + (x^2 + y^2)dy]$ , **04**  
 where  $C$  is the boundary in the  $xy$ -plane of the area enclosed by the  $x$ -axis and the semi-circle  $x^2 + y^2 + 1$  in the upper half  $xy$ -plane.
- (ii) Find a unit vector normal to the surface  $x^3 + y^3 + 3xyz = 3$  at the point  $(1,2,-1)$ . **03**

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