Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER 1st / 2nd EXAMINATION (OLD SYLLABUS) – SUMMER - 2017

Subject Code: 110014 **Date:01/06/2017**

Subject Name: Calculus

Time:2:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	1) Find the Taylor's series about $x = 0$ for $f(x) = \frac{1}{1+x^2}$.	**03**
		2) Find the expansion of $\tan\left(x + \frac{\pi}{4}\right)$ in ascending powers of x up to terms in x^3 and find approximately the value of $\tan 43^0$.	**04**
	(b)	1) Does the series $\sum_{n=1}^{\infty}(1 - e^{-n})$ converges ? Justify.	**03**
		2) Evaluate: i. $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$ ii. $\lim_{x \to 0} (\sin x)^{\tan x}$	**04**
Q.2	**(a)**	1) Determine whether the series $\sum_{n=1}^{\infty} \frac{(n-1)!}{n^2}$ converges or diverges.	**03**
		2) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty}(-1)^n \frac{x^{2n-1}}{(2n-1)!}$.	**04**
	(b)	1) Determine whether the series $\sum_{n=1}^{\infty} \frac{(n-1)}{n^3+3}$ converges or diverges.	**03**
		2) Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & ;(x,y) \neq (0,0) \\ 0 & ;(x,y) = (0,0) \end{cases}$ at $(0,0)$.	**04**
Q.3	**(a)**	1) Does the improper integral $\int_0^{\infty} e^{-5x} dx$ converge or diverge ? Justify.	**03**
		2) Compute the four second order partial derivatives of $f(x,y) = xy^2 + 3x^2e^y$.	**04**
	(b)	1) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$.	**03**
		2) Find the maximum and minimum values of $x + y$ on the circle $x^2 + y^2 = 4$.	**04**
Q.4	**(a)**	1) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 14$ at the point $(1,2,3)$.	**03**
		2) If $u = \cos^{-1} \frac{x^3+y^3}{x+y}$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\sin 2u \sin u - 4\cos u}{\sin^3 u}$.	**04**
	(b)	1) Determine the intervals of increasing and decreasing for the function $f(x) = x^3 - 12x - 5$.	**03**
		2) Evaluate $\iint_D (x + 2y) dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.	**04**

Q.5	**(a)**	1)	The region between the curve $y = \sqrt{x}$; $0 \leq x \leq 4$ and the $x - axis$ is revolved about the $x - axis$ to generate a solid. Find its volume.	**03**
		2)	Find the maximum and minimum values of the function $f(x) = x^3 + y^3 - 3x - 12y + 20$.	**04**
	(b)	1)	Evaluate $\int_0^1 \int_{x^2}^{x} (1 + xy) dydx$	**03**
		2)	Evaluate the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$ by reversing the order of integration.	**04**
Q.6	**(a)**		Trace the curve $xy^2 = 4a^2(2a - x)$; $a > 0$.	**07**
	(b)	1)	Evaluate the improper integral $\int_1^\infty \frac{1}{x^2} dx$.	**03**
		2)	Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} (x^2 + y^2)\, dydx$ by changing into polar coordinates where $a > 0$.	**04**
Q.7	**(a)**		Trace the curve $r = a(1 + cos\theta)$.	**07**
	(b)	1)	Evaluate $\int_0^4 \int_0^4 \int_0^4 (1 + xyz)\, dxdydz$.	**03**
		2)	Expand $e^x siny$ in powers of x and y up to second order term.	**04**

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