Seat No.: _____ **Enrolment No._____**

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) - EXAMINATION – SUMMER 2017

Subject Code: 130001 **Date: 25/05/2017**

Subject Name: Mathematics-3

Time: 10:30 AM to 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Obtain series solution of $\frac{d^2y}{dx^2} + y = 0$. **07**

(b) **Attempt any two of the following.** **07**

1) $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$

2) $xdy - ydx = \sqrt{x^2 + y^2}$

3) $\frac{dy}{dx} + y\cot x = \cos x$

4) $\sin(y - xp) = p$. Where, $p = \frac{dy}{dx}$.

Q.2 **(a)** Obtain the Frobenius series solution of $2x^2y'' + 3xy' - (x^2 + 1)y = 0$. **07**

(b) **Attempt any two of the following.** **07**

1) $\frac{d^4y}{dx^4} + 13\frac{d^2y}{dx^2} + 36y = 0$

2) $(D^2 + 3D + 2)y = 5$.

3) $y'' - 2y' + y = \cos 2x$

4) Solve by Method of variation of parameters. $y'' + a^2y = \tan ax$.

OR

(b) **Attempt any two of the following.** **07**

1) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2\log x$.

2) Using method of undetermined multipliers solve $y'' + 4y = 8x^2$.

3) Using method of undetermined multipliers solve $y'' - 3y' + 2y = e^x$.

4) Prove that $\int_0^\infty \frac{x^c}{c^x}dx = \frac{\Gamma(c+1)}{(\log c)^{c+1}}$

Q.3 **(a)** Define Laplace Transformation of a function $f(t)$ and using it obtain $L(\sin at)$ and $L(t^n)$. **07**

(b) **Attempt any two of the following.** **07**

1) Find the Laplace transform of $t^3 + \sin 2t + 5\cosh t$.

2) Evaluate $L(\sin t\sin 2t\sin 3t)$.

3) Evaluate $L\left[e^{-3t}(\cos 4t + 3\sin 4t)\right]$.

4) Evaluate $L^{-1}\left[\frac{1}{(s-1)(s^2-1)}\right]$.

OR

Q.3 **(a)** Define periodic function and obtain the Laplace transformation of periodic function having fundamental period p, **07**

(b) **Attempt any two of the following.** **07**

1) Evaluate $L\left(\frac{1-\cos 2t}{t}\right)$.

2) Evaluate $L\left(t^3 e^{-3t}\right)$.

3) Using convolution theorem evaluate $L^{-1}\left[\frac{1}{s^2\left(s^2+a^2\right)}\right]$.

4) Using Laplace transform technique solve the following IVP.

$y'' + 4y = \sin t \,\big|\, y(0) = 1,\ y'(0) = 0$.

Q.4 **(a)** Obtain Fourier series for the function $f(x) = x - x^2$ over $-\pi < x < \pi$ and hence show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{12}$. **07**

(b) **Attempt any one of the following.** **07**

1) Obtain the Fourier series for the function $f(x) = x^2, -\pi < x < \pi$. Hence show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

2) Find the Fourier series to represent the function $f(x)$ given by $f(x) = \begin{cases} x & \text{for } 0 \le x \le \pi. \\ 2\pi - x & \text{for } \pi \le x \le 2\pi. \end{cases}$ Hence show that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

3) Obtain half-range cosine series for the function $f(x) = \begin{cases} x & \text{for } 0 < x < \pi/2. \\ \pi - x & \text{for } \pi/2 < x < \pi. \end{cases}$

OR

Q.4 **(a)** Expand $f(x) = e^{-x}$ as a Fourier series in the interval $(-l, l)$. **07**

(b) **Attempt any two of the following.** **07**

1) Express the following function as Fourier integral $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$,

Hence evaluate a) $\int_0^{\infty} \frac{\sin \omega \cos \omega x}{\omega} d\omega$ and b) $\int_0^{\infty} \frac{\sin x}{x} dx$.

2) Show that $\frac{d}{dx}\left\{x^n J_n(x)\right\} = x^n J_{-n}(x)$.

3) Show that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$.

Q.5 **(a)** Solve the one-dimensional wave equation together with following initial & boundary conditions. **07**

$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. Where, $c^2 = T/\rho$.

$u(0,t) = u(l,t) = 0, \forall\ t > 0$

$u(x,0) = f(x)$ and $u_t(x,0) = g(x), \forall\ 0 < x < l$

(b) **Attempt any two of the following.** **07**

1) $xp + yq = 3z$

2) $(mz - ny)p + (nx - lz)q = (ly - mx)$

3) $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

OR

Q.5 **(a)** A homogenous rod of conducting material of length 100 cm. has its ends kept at zero temperature and the temperature initially is **07**

$$u(x,0) = \begin{cases} x & \text{for} \quad 0 \le x \le 50 \\ 100 - x & \text{for} \quad 50 \le x \le 100 \end{cases}$$

. Find the temperature $u(x,t)$ at any time *t*, at a distance *x*.

(b) **Attempt any two of the following.** **07**

1) $z = px + qy + \sqrt{1 + p^2 + q^2}$

2) $p^2 - q^2 = x - y$

3) $z = p^2 + q^2$

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