Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER 1st / 2nd EXAMINATION (NEW SYLLABUS) – SUMMER - 2017

Subject Code: 2110014 **Date:01/06/2017**

Subject Name: Calculus

Time: 2:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)** **Mark**

(a) Choose the appropriate answer for the following question. **07**

1. Infinite series $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots$ is _____ .

(A) Divergent (B) Convergent (C) Oscillation (D) None of these

2. The series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots$ represent expansion of _____.

(A) e^x (B) log (1 + x) (C) sin x (D) cos x

3. The value of the $\lim_{x\to 0}\left(\frac{\sin 2x}{x}\right) =$ _____.

(A) 2 (B) 1 (C) –1 (D) $\frac{1}{2}$

4. Asymptote parallel to y-axis of the curve $y = \frac{3x^2}{x-2}$ is the line _____.

(A) x = 0 (B) y = 0 (C) x = 2 (D) y = 2

5. f(x) = $|x|$ is _____ at origin.

(A) continuous (B) discontinuous (C) differentiable (D) None of these

6. Curve $y^2(a + x) = x^2(b – x)$ is symmetric about _____.

(A) x - axis (B) y - axis (C) line x = b (D) line x = –a

7. The curve increases strictly in the interval in which $\frac{dy}{dx}$ _____.

(A) < 0 (B) > 0 (C) = 0 (D) None of these

(b) Choose the appropriate answer for the following question. **07**

1. The values of $\int_0^\infty e^{-x}\cos(2x)\,dx$ = _____ .

(A) 0 (B) $-\frac{1}{5}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$

2. What does the region $\int_1^2\int_1^2 dxdy$ = _____ .

(A) rectangle (B) square (C) circle (D) triangle

3. The values of the $\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{x-\frac{\pi}{2}}\right) =$ _____.

(A) 0 (B) 1 (C) –1 (D) $\frac{\pi}{2}$

4. The function $f(x,y) = x^2y\, f(y/x)$ is homogeneous of degree is ____ .

(A) 0 (B) 1 (C) 2 (D) 3

5. The equation of the form $f(xy) = c$, then $\frac{dy}{dx}$ = _____ .

(A) $-\frac{f_x}{f_y}$ (B) $\frac{f_x}{f_y}$ (C) $-\frac{f_y}{f_x}$ (D) $\frac{f_y}{f_x}$

6. The values of $\int_0^1\int_0^1(3x^2 - 2y^2)dxdy$ is _____ .

(A) 0 (B) 1 (C) – 1 (D) $\frac{1}{3}$

7. If $x = u + 3v$, $y = v - u$ then the values of $\frac{\partial(x,y)}{\partial(u,v)}$ is _____.

(A) 4 (B) –1 (C) 5 (D) 7

Q.2 **(a)** Expand log(sec x) in power of x. **04**

(b) Evaluate $\lim_{x\to 0}\left(\frac{1^x + 2^x + 3^x}{3}\right)^{\frac{1}{x}}$. **03**

(c) (i) Trace the curve $y^2(2a - x) = x^3$. **04**

(ii) determine $\int_0^3 \frac{1}{\sqrt{3-x}}\, dx$ converge or diverges. **03**

Q.3 **(a)** If $f(x,y) = x^2y + xy^2$ then find $f_x(1,2)$ and $f_y(1,2)$ by definition. **04**

(b) Check the continuity for the following function at (0,0) **03**

$f(x,y) = \frac{2xy}{x^2+y^2}$, $(x,y) \neq (0,0)$

$= 0$, $(x,y) = (0,0)$

(c) (i) if $u = \sin^{-1}\left(\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}\right)$ then find the values of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ and **04**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}.$$

(ii) if $u = \frac{e^{x+y+z}}{e^x + e^y + e^z}$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 2u$. **03**

Q.4 **(a)** Find the extreme values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. **04**

(b) Find the equation of the tangent plane and normal line to the surface $2x^2 + y^2 + 2z = 3$ at (2, 1, –3) **03**

(c) (i) Find a point on the plane $2x + 3y - z = 5$ which is nearest to the origin. **04**

(ii) Expaned e^{xy} in power of $x - 1$ and $y - 1$ using Taylor's expansion. **03**

Q.5 **(a)** Test for the convergence the series; $\frac{1}{1.2.3} + \frac{x}{4.5.6} + \frac{x^2}{7.8.9} + \ldots\ldots$ **04**

(b) Test for the convergence the series; **03**

$$\sum_{n=1}^{\infty} [\sqrt[3]{n^3 + 1} - n]$$

(c) (i) Determine absolute or conditional convergence of the series; **04**

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3 + 1}$$

(ii) Test the convergence for $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots..$ **03**

Q.6 **(a)** Evaluate $\iint r\sqrt{a^2 - r^2}\, dr d\theta$ over the upper half of the circle $r = a\cos\theta$. **04**

(b) Sketch the region of integration and evaluate **03**

$$\int_1^4 \int_0^{\sqrt{x}} \frac{3}{2} e^{\frac{y}{\sqrt{x}}}\, dydx .$$

(c) (i) Evaluate the integral $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\, dxdy$ by changing the order of integration. **04**

(ii) Evaluate the integral $\int_0^2 \int_1^z \int_0^{yz} xyz\, dxdydz$. **03**

Q.7 **(a)** Find the area included between the curve $y^2(2a - x) = x^3$ and its asymptote. **04**

(b) Find the volume of a solid generated by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line. **03**

(c) Use triple integration to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ between the planes $z = 1$ and $x + z = 1$. **07**

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