**Seat No.: _____** **Enrolment No._____**

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–IV (NEW) - EXAMINATION – SUMMER 2017**

**Subject Code: 2140001** **Date: 30/05/2017**

**Subject Name: Mathematics-4**

**Time: 10:30 AM to 01:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** Answer each of the following questions. **14**

**1** Find the real and imaginary part of $f(z) = z^2 + 5z$.

**2** Define: domain

**3** Is $f(z) = \bar{z}$ analytic? Give the reason.

**4** Evaluate $\int_C \exp(z^4)\,dz$, Where C is any simple closed curve.

**5** What does conformal mapping mean?

**6** What are the fixed points of the mapping $w = \dfrac{(3iz + 13)}{(z - 3i)}$?

**7** What is the order of pole of the point $z = 0$ for the function $\dfrac{\sin z}{z^5}$?

**8** State the De Moivre's theorem.

**9** If $f(x) = \dfrac{1}{x}$, find the divided difference of $[x_0, x_1, x_2]$.

**10** State: Intermediate value property.

**11** Write the formula of Euler's method to find the solution of IVP:

$$\frac{dy}{dx} = f(x, y);\ y(x_0) = y_0$$

**12** Name two different iterative methods for the solution of a system of linear equations.

**13** State the Trapezoidal rule with $n = 12$.

**14** Write the formula for the quadratic Lagrange interpolation.

**Q.2** **(a)** Define a harmonic function. Show that $u(x, y) = x^2 - y^2$ is harmonic. Find a conjugate harmonic. **03**

**(b)** Define mobious transformation. Determine the mobious transformation which maps the points $0, \infty, i$ to $\infty, 1, 0$. **04**

**(c)** (i) Discuss the convergence of $\sum_{n=0}^{\infty} \dfrac{(2n)!(z - 2i)^n}{(n!)^2}$ and also find the radius of convergence. **04**

(ii) Expand $\log(1 + z)$ about $z = 0$ as a Taylor's series. **03**

**OR**

**(c)** Expand $f(z) = \dfrac{(z - 2)(z + 2)}{(z + 1)(z + 4)}$ in the region: **07**

(i) $|z| < 1$ (ii) $1 < |z| < 4$ (iii) $|z| > 4$

**Q.3** **(a)** Evaluate $\int_C |z|\,dz$, where C is the left half of the unit circle $|z| = 1$ from $z = -i$ to $z = i$. **03**

**(b)** State the Cauchy's integral theorem and Cauchy's integral formula. Using them evaluate $\int_C \left(\frac{e^z}{z-2}\right) dz$ where C is the circle (i) $|z| = 3$ , (ii) $|z| = 1$ **04**

**(c)** State Cauchy's residue theorem. Using it evaluate $\int_C \left[\frac{z^2}{(z-1)^2(z+2)}\right] dz$ where C is the circle $|z| = 4$. **07**

**OR**

**Q.3** **(a)** Find the residue of $f(z) = \frac{1-e^z}{z^4}$ at $z = 0$. **03**

**(b)** Define singular points of $f(z)$. Find the singular points of: **04**

$f(z) = \frac{z^2}{(z-1)(z-2)^2}$ and classify them.

**(c)** Using residues to evaluate $\int_0^\infty \frac{x^2\,dx}{(x^2+9)(x^2+4)^2}$. **07**

**Q.4** **(a)** Explain Newton-Raphson method for solving $f(x) = 0$ . **03**

**(b)** Apply Newton-Raphson method to find the solution of $\sin x = e^{-x}$ upto three decimal places, starting with $x_0 = 0.6$. **04**

**(c)** Compute a 7D (seven decimal)-value of the Bessel function $J_0(x)$ for $x = 1.72$ from four values in the following table, using **07**
(i) Newton's forward formula and
(ii) Newton's backward formula

| $x$ | 1.7 | 1.8 | 1.9 | 2.0 |
|---|---|---|---|---|
| $J_0(x)$ | 0.3979849 | 0.3399864 | 0.2818186 | 0.2238908 |

**OR**

**Q.4** **(a)** Evaluate $\int_0^1 \frac{dx}{1+x}$ by Gaussian integration for three points. **03**

**(b)** Evaluate $\int_1^2 e^{-x/2}\,dx$ using Simpson's 1/3 rule with h=0.25. **04**

**(c)** Is the coefficient matrix of the following linear equations diagonally dominant? If not, convert it into diagonally dominant and solve by Gauss-Seidel method. **07**
$10x + y + z = 6,\ \ x + y + 10z = 6,\ \ x + 10y + z = 6.$

**Q.5** **(a)** Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using the trapezoidal rule. (take h=1) **03**

**(b)** Compute $f(9.2)$ by using Langrage interpolation method from the following data **04**

| $x$ | 9 | 9.5 | 11 |
|---|---|---|---|
| $f(x)$ | 2.1972 | 2.2513 | 2.3979 |

**(c)** Apply fourth order Runge-Kutta method to find $y(0.1)$ and $y(0.2)$ given that $y' = x + y,\ \ y(0) = 1$ (take h=0.1) **07**

**OR**

**Q.5** **(a)** Find the smallest positive root of the equation $x^3 - 4x - 9 = 0$, using the bisection method in four stages. **03**

**(b)** Solve using partial pivoting by Gauss elimination method: **04**

$2x + 2y + z = 6,\ 4x + 2y + 3z = 4,\ x + y + z = 0.$

**(c)** Determine the largest eigenvalue and the corresponding eigenvector of the matrix $\mathbf{A}$ , **07**

Where $\mathbf{A} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ by power method.

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