

Seat No.: _____

Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY**BE (SPFU) - SEMESTER-I-II (SPFU) - EXAMINATION – SUMMER 2017****Subject Code: MTH001****Date: 29/05/2017****Subject Name: CALCULUS****Time:02:30 PM to 05:30 PM****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1	(a) Discuss Convergence of Sequence $i) \left(1 + \frac{1}{n}\right)^n$ $ii) \left(\frac{n!}{n^n}\right)$.	07
	(b) Discuss Convergence of Series $i) \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$ $ii) \sum_{n=1}^{\infty} \frac{8 \tan^{-1} n}{1+n^2}$.	07
Q.2	(a) State Ratio test for Series. Use it to discuss convergence of Series $\sum_{n=1}^{\infty} \frac{(n+3)!}{3! n! 3^n}$.	07
	(b) Discuss Convergence of power series $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$.	07
Q.3	(a) Use chain rule to find $\frac{\partial w}{\partial r}$ when $r = 1, s = -1$ if $w = (x+y+z)^2, x = r-s, y = \cos(r+s), z = \sin(r+s)$.	07
	(b) If $u = \sin^{-1} \frac{x^3 + y^3}{x + y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$.	07
Q.4	(a) Find equations for the tangent plane and normal line of surface $z^2 - 2x^2 - 2y^2 - 12 = 0$ at the point $P(1, -1, 4)$.	07
	(b) Find local extreme values of $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$.	07
Q.5	(a) Expand e^{xy} at $(1,1)$ in powers of $(x-1)$ and $(y-1)$.	07
	(b) Evaluate: $\int_0^1 \int_0^{y^2} 3y^3 e^{xy} dx dy$.	07
Q.6	(a) Change the order of integration and evaluate the given integral $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} dy dx$.	07
	(b) Evaluate $\iint_R (x-y)^4 e^{x+y} dx dy$, where R is the square with vertices $(1,0), (2,1), (1,2)$ and $(0,1)$.	07
Q.7	(a) Sketch the curve $y = x + \sin x, 0 \leq x \leq 2\pi$.	07
	(b) Sketch the curve $y = x^2 - 1 $.	07
