Subject Code: 110008

Subject Name: Maths-I

Date: 10-06-2013

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER- 1st / 2nd • EXAMINATION - SUMMER 2013

Time: 02:30 pm - 05:30 pm**Total Marks: 70 Instructions:** 1. Attempt any five questions. 2. Make suitable assumptions wherever necessary. 3. Figures to the right indicate full marks. (a) If $3x \le f(x) \le x^3 + 2$ for $0 \le x \le 2$, Evaluate $\lim_{x \to 1} f(x)$. 02 (b) Find the constant c that makes f continuous on $(-\infty, \infty)$, $f(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx - 20 & \text{if } x \ge 4 \end{cases}$ (c) Express the polynomial $x^3 + 7x^2 + x - 6$ in power of (x - 1). 03 03 (d) Evaluate: 06 1. $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}.$ $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}.$ O.2(a) Find the absolute maximum and absolute minimum values of 04 $f(x) = 3x^2 - 12x + 5$ on the interval [0,3]. (b) Expand $e^{\sin x}$ by Maclaurins series up to the terms containing x^4 . 05 (c) Find two positive numbers whose product is 100 and whose sum is minimum. 05 (a) If $f(x) = x^3 + x^2y^3 - 2y^2$, find $f_x(2,1)$ and $f_y(2,1)$. (b) If $u = \sin^{-1} \frac{x+2y}{x^8+y^8}$, find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$. 04 Q.3 05 (c) Find the local extreme values of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$. 05 0.4 (a) For what values of a, m and b does the function 04 $f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$ Satisfies the hypotheses of the Mean Value theorem on the interval [0,2]? (b) Find $\frac{d}{dx} \int_{1}^{x^{2}} \cos t \, dt$. (c) Determine whether series converges or diverges. $\begin{array}{ccc}
& \infty & \infty \\
& 1. & \Sigma & \frac{1}{n^{2}+n+1}. & 2. & \Sigma & \left(\frac{2n+3}{3n+12}\right)^{n}.\\
& n = 1 & n = 1
\end{array}$ (a) Evaluate: $\int_{1}^{2} \int_{y}^{2} xy \, dx \, dy$. 04 06 04 **(b)** Evaluate: $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2) dz dx dy$. 05 (c) Show that the area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $\frac{16}{3}a^2$. 05 **Q.6** (a) Find the jacobian of the transformation x = u + 4v and y = 3u - 2v. 04 05 (b) Evaluate the integral $\int_c xy \, dy - y^2 \, dx$, where c is the square cut from the first quadrant by the lines x = 1 and y = 1. 05 (c) If $\vec{f} = 3xyi - y^2j$, evaluate $\int \vec{f} \cdot dr$, where c is the curve in the xy-plane $y = 2x^2$ from (0,0) to (1,2).

Q.7 (a) Find the area enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. 04

(b) Evaluate: 04

- 1. $div [3x^2i + 5xy^2j + xyz^3k]$. 2. $curl[e^{xyz}(i+j+k)]$.
- (c) Verify Stoke's theorem for $\vec{f} = (x^2 + y^2) i 2xy j$ taken around the rectangle bounded by the lines $x = \pm a$, y = 0, y = b. 06
