Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III • EXAMINATION – SUMMER 2013

Subject Code: 130001 **Date: 01-06-2013**

Subject Name: Mathematics-III

Time: 02.30 pm - 05.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as directed **14**

(a) Find the Laplace transform of $t^2 \sin 2t$

(b) Evaluate $\beta\left(\frac{7}{2},\frac{5}{2}\right)$

(c) Solve $e^x \tan y dx + \left(1-e^x\right)\sec^2 y dy = 0$

(d) Find general solution of $y'''-y=0$

(e) Define Convolution and unit step function.

(f) Discuss singularities of $x^3(x-1)y''-3(x-1)y'+xy=0$

(g) Express $f(x)=3x^2+3x+1$ in terms of Legendre polynomial

Q.2 **(a)** By power series method solve $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$ **07**

(b) (i) Prove that $\frac{d}{dx}\left[x^n J_n(x)\right]=x^n J_{n-1}(x)$ **04**

(ii) Define Gamma function and Evaluate $\int_0^\infty x^6 e^{-2x}$ **03**

OR

(b) (i) Solve $x\frac{dy}{dx}+(1+x)y=x^3$ **03**

(ii) Prove that $(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x)$ **04**

Q.3 **(a)** Define Laplace transform and find Laplace transform of **07**

(i) $t^3+e^{-3t}+t^{1/2}$ (ii) $e^{-2t}\sin^2 2t$

(b) Find inverse laplace transform of **07**

(i) $\frac{3s^2+2}{(s+1)(s+2)(s+3)}$ (ii) $\frac{s^3+2s^2+2}{s^3(s^2+1)}$

OR

Q.3 **(a)** State and prove convolution theorem **07**

(b) Find inverse laplace transform of **07**

(i) $\log\left(\frac{s+1}{s-1}\right)$ (ii) $\frac{e^{-4s}(s+2)}{s^2+4s+5}$

Q.4 **(a)** (i) Using method of variation of parameter solve $y''-2y'+y=e^x x^{\frac{3}{2}}$ **04**

(ii) Solve $\left(D^2+D-6\right)y=e^{2x}\sin 3x$ **03**

(b) (i) Find the Fourier series of $f(x)=x+|x|$, where $x\in(-\pi,\pi)$ **04**

(ii) find Fourier series of $f(x)=x^3$ where $x\in(-\pi,\pi)$ **03**

OR

Q.4	**(a)**	(i) Solve $x^2y''+4xy'-4y=\sin(\ln x)$	**04**
		(ii)Using method of undetermined coefficients solve $y''-2y'+y=e^x+x$	**03**
	(b)	Solve using Frobenius method $2x(1-x)y''+(1-x)y'+3y=0$	**07**
Q.5	**(a)**	Using method of separation of variables solve $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+2u$; $u=0,\ \frac{\partial u}{\partial x}=1+e^{-3y}$ when $x=0$	**07**
	(b)	A rod 30cm long has its end A and B kept $20°C$ and $80°C$ respectively until steady state condition prevail. The temperature at each end is suddenly reduce to $0°C$ and kept so. Find the resulting temperature function $u(x,t)$ from end A.	**07**

OR

Q.5	**(a)**	Find the Fourier transform of e^{-ax^2} where $a>0$.	**07**
	(b)	Find Fourier cosine integral of $f(x)=\begin{cases} x & if\ 0<x<a \\ 0 & if\ x>a \end{cases}$	**07**

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