Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III • EXAMINATION – SUMMER 2013

Subject Code: 130002 **Date: 01-06-2013**

Subject Name: Advanced Engineering Mathematics

Time: 02.30 pm - 05.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	(i)	Solve $\left(1+y^2\right)\frac{dx}{dy}=\tan^{-1}y-x$.	**03**
		(ii)	Solve $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$.	**04**
	(b)		Find power series solution of the of the equation $\frac{d^2y}{dx^2}+xy=0$ in powers of x.	**07**
Q.2	**(a)**		Solve $x\frac{\partial u}{\partial x}-2y\frac{\partial u}{\partial y}=0$ using method of separation of variables.	**07**
	(b)		Find the power series solution of the equation $\left(x^2+1\right)y''+xy'-xy=0$ about $x=0$.	**07**
			OR	
	(b)		Solve $\left(D^3-D^2-6D\right)y=x^2+1$	**07**
Q.3	**(a)**		Find the fourier series of the periodic function $f(x)$ with period 2π, defined as follows: $f(x)=\begin{cases}0, & for \quad -\pi<x\le 0\\ x, & for \quad 0\le x<\pi\end{cases}$	**07**
	(b)		Find the fourier cosine series for $f(x)=x^2,\quad 0<x\le c$. Also sketch $f(x)$.	**07**
			OR	
Q.3	**(a)**		Find the fourier series of periodic function with period 2, which is given below. $f(x)=\begin{cases}0 & -1\le x\le 0\\ x & 0\le x\le 1\end{cases}$	**07**
	(b)		Find the fourier series of the periodic function with period 2 of $f(x)=\begin{cases}\pi & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$	**07**
Q.4	**(a)**	(i)	Prove that $L\left(e^{-at}\right)=\frac{1}{s+a},\quad s>-a$	**03**
		(ii)	Prove that $L\left(t^n\right)=\frac{n!}{s^{n+1}},\quad n$ being positive integer.	**04**
	(b)	(i)	Find $L^{-1}\left\{\log\frac{s+a}{s+b}\right\}$	**04**

		(ii)	Find $L^{-1}\left\{\dfrac{s+2}{\left(s^2+4s+5\right)^2}\right\}$	**03**

OR

Q.4	**(a)**		If $\overline{f(s)}$ is the laplace transform of $f(t)$ and $a \geq 0$, then prove that $L[f(t-a)u(t-a)] = e^{-as}\,\overline{f(s)}$	**07**
	(b)	(i)	Find the laplace transform $L\left[\int_0^t e^{-x}\cos x\,dx\right]$	**03**
		(ii)	Find the laplace transform $L\left[\int_0^t\int_0^t \sin au\,du\,du\right]$	**04**
Q.5	**(a)**	(i)	Define Beta function and Rectangle function.	**04**
		(ii)	Form partial differential equation $z = (x-2)^2 + (y-3)^2$	**03**
	(b)		Find the fourier transform of f defined by $f(t) = e^{-\lvert t\rvert}$. Sketch the graph.	**07**

OR

Q.5	**(a)**	(i)	Eliminate the function f from the relation $f\left(xy+z^2, x+y+z\right) = 0$.	**04**
		(ii)	Solve $\dfrac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\dfrac{\partial z}{\partial y} = -2\sin y$, when $x = 0$, and $z = 0$, when y is an odd multiple of $\dfrac{\pi}{2}$.	**03**
	(b)	(i)	Solve $x(y-z)p + y(z-x)q = z(x-y)$	**04**
		(ii)	Solve $z = px + qy + p^2q^2$	**03**

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