Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV • EXAMINATION – SUMMER 2013

Subject Code: 140001 **Date: 05-06-2013**

Subject Name: Mathematics - IV

Time: 10.30 am - 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find the real and imaginary part of $f(z) = z^2 + 3z$. **02**

(b) Sketch the region $|z| \le 1$. Is it a domain? **02**

(c) For the principle branch show that $\text{Log}(i^3) \neq 3\,\text{Log}(i)$. **02**

(d) Evaluate $\oint_C (z^2 + 3)dz$ where C is any closed contour. Justify your answer. **02**

(e) Prove that $\sin^{-1} z = -i \ln(iz + \sqrt{1 - z^2})$ **02**

(f) Prove that $E = 1 + \Delta$ where Δ is forward deference and E is shift operator. **02**

(g) Discuss the singularity of the point z = 0 for the function $\frac{\sin z}{z}$. **02**

Q.2 **(a)** Explain Newton's method for solving equation f(x) = 0. Apply this method to find the approximate solution of $x^3 + x - 1 = 0$ correct up to three decimal. **07**

(b) Write the trapezoidal rule for numerical integration. Using Simpson's 1/3 rule evaluate $\int_1^{2.5} f(x)dx$ from the following data. Take h = 0.3. **07**

x	1	1.3	1.6	1.9	2.2	2.5
f(x)	1	1.69	2.56	3.61	4.84	6.25

OR

(b) Write formula for Range Kutta method for order four. Apply Euler's method to find the approximate solution of $\frac{dy}{dx} = x + y$ with y(0) = 0 and h = 2. Show your calculation up to five iterations. **07**

Q.3 **(a)** Explain quadratic Langrage interpolation. Compute f(9.2) by using Langrage interpolation method from the following data. **07**

x	9	9.5	11
f(x)	2.1972	2.2513	2.3979

(b) Use Newton's forward deference method to find the approximate value of f(1.3) from the following data **07**

x	1	2	3	4
F(x)	1.1	4.2	9.3	16.4

OR

Q.3 **(a)** Write a formula for divided difference $f[x_0, x_1]$ and $f[x_0, x_1, x_2]$. Using Newton's divided difference formula compute f(10.5) from the following data **07**

x	10	11	13	17
F(x)	2.3026	2.3979	2.5649	2.8332

(b) Use Gauss Seidel method to determine roots of the following simultaneous equations. **07**

$$2x - y = 3$$
$$x + 2y + z = 3$$
$$-x + z = 3$$

Q.4 **(a)** Define a harmonic function. Show that $u(x, y) = x^2 - y^2$ is harmonic. Find the corresponding analytic function $f(z) = u(x,y) + iv(x,y)$. **07**

(b) Define a linear fractional transformation (*Möbius* transformation). Find the bilinear transformation that maps the points $z_1 = -1$, $z_2 = 0$, $z_3 = 1$ onto $w_1 = -i$ $w_2 = 1$, $w_3 = i$ respectively. **07**

Also find w for $z = \infty$.

OR

Q.4 **(a)** State de Moivre's formula. Find and graph all fifth root of unity in complex plane. **07**

(b) State Liouville's theorem and Maximum Modulus theorem. Without using integration show that $\left|\oint_C \frac{e^z}{z+1} dz\right| \le \frac{8\pi e^4}{3}$ where C is $|z| = 4$ **07**

Q.5 **07**

(a) Evaluate (i) $\int_0^{2+i} z^2 dz$ along the line y = x/2

(ii) $\oint_C \frac{5z+7}{z^2+2z-3} dz$ where C is $|z-2| = 2$

(b) Find Laurent's series expansion in power of z that represent $f(z) = \frac{1}{z^2(1-z)}$ for domain **07**

(i) $|z| < 1$ and (ii) $|z| > 1$

OR

Q.5 **(a)** (i) Evaluate $\oint_C \tan z dz$ where C is $|z| = 2$ **07**

(ii)Evaluate $\oint_C \frac{2z+6}{z^2+4} dz$ C is $|z-i| = 2$

(b) Evaluate a real integral $\int_0^{2\pi} \frac{1}{(2+\cos\theta)^2} d\theta$ using residue. **07**

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