

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER- 1st / 2nd • EXAMINATION – SUMMER • 2014

Subject Code: 110009

Date: 16-06-2014

Subject Name: Mathematics - II

Time: 02:30 pm - 05:30 pm

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Find the inverse of $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ 07
- (b) What conditions must $b_1, b_2,$ and b_3 satisfy in order for the system of equations $x + y + z = b_1, x + z = b_2, 2x + y + 3z = b_3$ to be consistent? 07
- Q.2** (a) Find the eigen values and one of the eigen vector of the matrix 07
- $$A = \begin{bmatrix} 8 & 5 & 0 \\ 0 & 3 & 0 \\ -9 & -1 & -1 \end{bmatrix}$$
- (b) Find the basis for the null space of the matrix 07
- $$A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$
- Q.3** (a) Use the inner product $\langle p, q \rangle = \int_{-1}^1 p(x)q(x)dx$.To compute $\langle p, q \rangle$ 07
for the vectors $p = p(x)$ and $q = q(x)$ in P_3 where
1. $p = 1 - x + x^2 + 5x^3$ $q = x$
 2. $p = x - 5x^3$ $q = 2 + 8x^2$
- (b) Find the orthonormal basis for the subspace spanned by 07
 $\{ (1,1,1), (1,-2,1), (1,2,3) \}$.
- Q.4** (a) Consider the basis $S = \{ v_1, v_2, v_3 \}$ for \mathbb{R}^3 , where $v_1 = (1,1,1)$, $v_2 = (1,1,0)$ and $v_3 = (1,0,0)$. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation such that $T(v_1) = (1,0)$ 07
 $T(v_2) = (2,-1)$ and $T(v_3) = (4,3)$. Find a formula for $T(x, y, z)$ and compute $T(2,1,3)$
- (b) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation defined by 07
 $T(x, y) = (y, -5x + 13y, -7x + 16y)$. Find the matrix for the linear transformation T with respect bases $B = \{ b_1 = (3,1) b_2 = (5,2) \}$ for \mathbb{R}^2 and $B' = \{ v_1 = (1,0,-1), v_2 = (-1,2,2), v_3 = (0,1,2) \}$.
- Q.5** (a) Solve the following system of the equations 07
 $x + 2y + z = 5, 3x - y + z = 6, x + y + 4z = 7$.
- (b) Define: symmetric and skew-symmetric matrix. Show that every square matrix can be expressed as sum of symmetric and skew-symmetric matrix. 07
- Q.6** (a) Consider the vectors $u = (2,-1,1)$ and $v = (1,1,2)$ 04
Find 1. $u + v$ 2. $u \cdot v$ 3. $\|u - v\|$ and determine the angle between u and v .
- (b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ 04

- (c) Let $A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$. Compute A^3 , A^{-3} and $A^2 - 2A + I$. **06**
- Q.7** (a) Show that the vectors $v_1 = (1, 2, 3)$, $v_2 = (4, 5, 6)$ and $v_3 = (2, 1, 1)$ in \mathbb{R}^3 linearly independent. **04**
- (b) Define: Basis for Vector Space. **04**
Show that the set of vectors $S = \{ (1,2,1), (2,9,0), (3,3,4) \}$ is a basis for \mathbb{R}^3 .
- (c) Determine which of the following are subspace of \mathbb{R}^3 **06**
1. $W = \{ (x,y,0) / x, y \in \mathbb{R} \}$
 2. $U = \{ (x, 1, 1) / x \in \mathbb{R} \}$.
