Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III • EXAMINATION – SUMMER • 2014

Subject Code: 130002 Date: 02-06-2014

Subject Name: Advanced Engineering Mathematics

Time: 02.30 pm - 05.30 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ **03**

(b) $[(x+1)e^x - e^y]\,dx - x e^y\,dy = 0,\ y(1) = 0.$ **04**

(c) Determine the series solution for the differential equation $y'' + y = 0$ about $x_0 = 0.$ **07**

Q.2 **(a)** (i) Solve $(D^2 + 5D + 6)y = e^x$. **03**

(ii) Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by method of variation of parameter. **04**

(b) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variable. **07**

OR

(b) Solve in series the equation $\frac{d^2y}{dx^2} + xy = 0$. **07**

Q.3 **(a)** Find the Fourier series for $f(x) = e^{-x}$, $0 < x < 2\pi$. **07**

(b) Find the Fourier series expansion for $f(x)$, *if* $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ **07**

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots = \frac{\pi^2}{8}$.

OR

Q.3 **(a)** Find the Fourier series expansions of **07**

(i) $f(x) = x$, $-\pi < x < \pi$ $\qquad f(x + 2\pi) = f(x)$

(ii) $f(x) = x^2$ $-l < x < l$.

(b) Express $f(x) = x$ as a **07**

(i) half range sine series in 0 < x <2

(ii) half range cosine series in 0 < x < 2.

Q.4 **(a)** (1) Prove that $L(\sinh at) = \frac{a}{s^2 - a^2}$ *for* $s > |a|$. **03**

(2) Find the Laplace transforms of (i) sin2tsin3t (ii) $e^{-3t}(2\cos 5t - 3\sin 5t)$ **04**

(b) Evaluate : (i) $L^{-1}\{\ln(1 + \frac{w^2}{s^2})\}$ (ii) $L^{-1}\{\frac{5s+3}{(s-1)(s^2+2s+5)}\}$ **07**

OR

Q.4 **(a)** Apply convolution theorem to evaluate $L^{-1}\left(\frac{s}{(s^2 + a^2)^2}\right)$ **03**

(b) Use Laplace transform method to solve $y'' + a^2 y = k \sin at$ **04**

(c) If $L(f(t)) = \overline{f(s)}$ and if $L\left\{\frac{f(t)}{t}\right\}$ exists then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \overline{f}(s)\, ds$. Also find $L\left\{\frac{\sin 2t}{t}\right\}$ **07**

Q.5 **(a)** (1) Form partial differential equation of $f(x+y+z, x^2+y^2+z^2) = 0$, where f is an arbitary function. **03**

(2) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ **04**

(b) (1) Solve $p(1+q) = qz$ **07**

(2) Solve $p^2 + q^2 = x + y$

OR

Q.5 **(a)** Define following terms (i) Beta function (ii) Sinusoidal function **04**

(b) Form partial differential equation $z = (x-2)^2 + (y-3)^2$ **03**

(c) Find the Fourier integral representation of the function $f(x) = \begin{cases} 2, & |x| < 2 \\ 0, & |x| > 2 \end{cases}$ **07**

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