Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER– 1st /2nd (OLD SYLLABUS) EXAMINATION – SUMMER 2015

Subject Code: 110008 **Date: 02/06/2015**

Subject Name: Mathematics-1

Time: 10.30am-01.30pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	(i)	Verify Cauchy's mean value theorem for $\frac{1}{x}$ and $\frac{1}{x^2}$ $\forall$ $x \in$[a,b], a>0.	**03**
		(ii)	Find maxima and minima of the function f(x,y)=$x^3+y^3-3x-12y+20$.	**04**
	(b)	(i)	Find the Taylor's series expansion f(x,y)=x^3-2x+4 at a=2.	**03**
		(ii)	Evaluate using L'Hospital rule $\lim_{x\to 0} \frac{e^x-1-x}{x^2}$	**02**
		(iii)	Evaluate using L'Hospital rule $\lim_{x\to 1}$ (1-x) $\tan(\frac{\pi x}{2})$	**02**
Q.2	**(a)**	(i)	Trace the curve $1 \le r \le 2$, $0 \le \theta \le \frac{\pi}{2}$	**04**
		(ii)	Discuss convergence of the integral $\int_0^4 \frac{1}{x^{-2}}dx$.	**03**
	(b)	(i)	Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.	**04**
		(ii)	State fundamental theorem of integral calculus and hence find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t\,dt$	**03**
Q.3	**(a)**	(i)	Test the convergence of $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$.	**04**
		(ii)	Does sequence whose n^{th} term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ converges? If so find, limit of a_n.	**03**
	(b)	(i)	Show that the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+.....$ is convergent for p > 0.	**04**
		(ii)	Find convergence of sequence $\left\{\frac{(-1)^n}{2}\right\}$.	**03**
Q.4	**(a)**	(i)	State and prove Euler's theorem on homogenous functions.	**04**

		(ii)	If u = $\text{cosec}^{-1}\left(\frac{x+y}{x^2+y^2}\right)$, show that $xu_x + yu_y = \tan u$.	**03**
	(b)	(i)	Find the equations of tangent plane and normal line to the surface $2x^2+y^2+2z=3$ at the point (2,1,-3).	**04**
		(ii)	The region between the curve $y=\sqrt{x}$, $0 \le x \le 4$ and X-axis is revolved about X-axis is generate a solid then find volume.	**03**
Q.5	**(a)**	(i)	Evaluate $\int_0^1 \int_y^{1+y^2} x^2 y\,dx\,dy$	**04**
		(ii)	If $x = r\cos\theta, y = r\sin\theta, z = z$, calculate $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$	**03**
	(b)	(i)	Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$	**04**
		(ii)	If $u=2xy, v=x^2-y^2,\ x = r\cos\theta, y = r\sin\theta, z = z$, calculate $\frac{\partial(u,v)}{\partial(r,\theta)}$	**03**
Q.6	**(a)**	(i)	Find the direction derivative of $f(x,y,z) = 4xz^3-3x^2y^2z$ at the point (2,-1,2).	**04**
		(ii)	Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point (2,-1,3).	**03**
	(b)	(i)	Evaluate using Green's theorem, $\int_c \left[(x^2+xy)dx + (x^2+y^2)dy\right]$ where c is the surface bounded by x = ±1, y = ±1.	**04**
		(ii)	Prove that vector $\vec{F} = (z+\sin y)i + (x\cos y - z)j + (x-y)k$ is irrotational vector.	**03**
Q.7	**(a)**	(i)	By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.	**04**
		(ii)	Find $\frac{dy}{dx}$ for $(\cos x)^y = x^{\sin y}$ using partial differentiation.	**03**
	(b)	(i)	Does limit of function $\frac{2xy}{x^2+3y^2}$ exists at x=0, y=0?	**04**
		(ii)	Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n\log n}$.	**03**

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