Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III EXAMINATION – SUMMER 2015

Subject Code:130001 **Date:06/06/2015**

Subject Name:Mathematics-III

Time:02.30pm-05.30pm **TotalMarks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Find the General Solution of the differential equation $y' = e^{x-y} + xe^{-y}$. **02**

(ii) Find the Particular Solution of the Differential Equation $y'' + 9y = \cos 5x$ by using Method of Undetermined Co-efficients. **02**

(iii) Find the Inverse Laplace Transform of following function: **03**

$$\frac{1}{\left(s+\sqrt{2}\right)\left(s-\sqrt{3}\right)}$$

(b) (i) Define Ordinary Point of the differential equation $y'' + P(x)y' + Q(x)y = 0$. **02**

(ii) Find the value of $\beta(2,3)$ **02**

(iii) Express the function $f(x) = x^3$ as a Fourier Series in interval $[-\pi, \pi]$. **03**

Q.2 **(a)** (i) Evaluate: $\int_0^\infty x^2 e^{-x^3} dx.$ **02**

(ii) Solve: $\left(D^3 - 1\right)y = 0.$ **02**

(iii) Solve the Partial Differential Equation: $u_{xy} = \cos x \cos y.$ **03**

(b) (i) Find the Laplace Transform of function $f(t) = 100^t + 2t^{10} + \sin 10t$ **03**

(ii) Using Method of Variation of Parameters solve the differential equation $y'' + y = \cot x.$ **04**

OR

(b) (i) Find the Laplace Transform of the function $f(t) = t \sin t.$ **03**

(ii) Solve: $\left(D^2 - 49\right)y = \sinh 3x$ **04**

Q.3 **(a)** Find the Laplace Transforms of following functions: **07**

(i) $\sin^3 2t$ (ii) $\sin^2 2t$.

(b) State Convolution Theorem and using it find Inverse Laplace Transform of function $F(s) = \dfrac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$ **07**

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OR

Q.3 **(a)** Using Laplace Transform solve the differential equation: **07**

$y''-4y'+3y=6t-8,\ y(0)=0,\ y'(0)=0$.

(b) **07**

Evaluate: (i) $\beta(m,n)=2\int_0^{\frac{\pi}{2}}\sin^{2m-1}\theta\cos^{2n-1}d\theta.$

(ii) $\int_0^{\frac{\pi}{2}}\sqrt{\tan\theta}\,d\theta.$

Q.4 **(a)** (i) Prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ **03**

(ii) $P_n(1)=1$. **04**

(b) (i) Solve the differential equation $y''+y=0$ by the Power Series Method. **03**

(ii) State Rodrigue's Formula and using it compute $P_2(x)$. **04**

OR

Q.4 **(a)** (i) Solve the differential equation $(x+y)dx+(y-x)dy=0.$ **03**

(ii) Solve : $x^2\frac{d^2y}{dx^2}-2y=x^2+\frac{1}{x}.$ **04**

(b) (i) If $y_1=e^{-2x}$ is One Solution of $y''+4y'+4y=0$ then find the Second Solution. **03**

(ii) Solve: $(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=3x^2+4x+1.$ **04**

Q.5 **(a)** (i) Find Half Range Cosine Series for the function $f(x)=e^{ax}$ in interval $[0,1]$. **03**

(ii) Express the function $f(x)=x+x^2$ as a Fourier Series in interval $[-\pi,\pi]$. **04**

(b) **03**

(i) Evaluate: $\int_0^1\frac{1}{\sqrt{1-x^4}}dx.$

(ii) By using the relation between Beta and Gamma function prove that **04**

$$\frac{\beta(m,n+1)}{n}=\frac{\beta(m+1,n)}{m}=\frac{\beta(m,n)}{m+n}.$$

OR

Q.5 **(a)** Solve completely the equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ representing the vibrations of a string of length l fixed at both ends given that, **07**

$$y(0,t)=y(l,t)=0,\ y(x,0)=f(x),\ \frac{\partial y}{\partial t}(x,0)=0,\ 0<x<l.$$

(b) Find the Fourier Transform of the function f defined as follows: **07**

$$f(x)=\begin{cases} xe^{-x} & x>0; \\ 0 & x<0. \end{cases}$$

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