Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2015

Subject Code: 2140001 **Date:26/05/2015**

Subject Name: MATHEMATICS-IV

Time: 10:30am-1.30pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) State the necessary and sufficient condition for a function to be analytic and prove the necessary condition. **[05]**

(b) Attempt the following. **[09]**

1. If α and β are the roots of equation $x^2-2x+4=0$ then show that $\alpha^{n+1}+\beta^{n+1}=2^{n+1}\cos\frac{n\pi}{3}$.

2. Determine the analytic function, $w=u+iv$ if $v=\log\left(x^2+y^2\right)+x-2y$.

3. Find and plot all the roots of $(8i)^{1/3}$.

Q.2 (a) Attempt the following.

1. Define bilinear transformation. Also find the bilinear transformation which maps the points $z=1,\ i,-1$ into the points $w=i,\ 0,\ -i$. Hence find the image of $|z|<1$. **[05]**

2. Evaluate $\oint_C \frac{e^{-z}}{z+1}dz$, Where C is the circle a) $|z|=2$. b) $|z|=1/2$. **[03]**

(b) Attempt the following. **[06]**

1. Find the image of infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=\frac{1}{z}$. Also show the region graphically.

2. Define residue at simple pole and find the sum of residues of the function $f(z)=\frac{\sin z}{z\cos z}$ at its poles inside the circle $|z|=2$.

OR

(b) Attempt the following. **[08]**

1. State Cauchy's integral formula and hence evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$; where C is the circle $|z|=2$.

2. Expand $f(z)=-\frac{1}{(z-1)(z-2)}$ in the region

a) $|z|<1$ **b)** $1<|z|<2$ **c)** $2<|z|<\infty$

Q.3 (a) State and prove Cauchy's integral theorem. **[05]**

(b) Attempt the following. **[09]**

1. Write the two Laurent series expansion in powers of z that represent the function $f(z)=\frac{1}{z^2(1-z)}$ in certain domains, and also specify domains.
2. Use Bisection method to find the real root of the equation $x^3-4x-9=0$ correct to two decimal places.
3. Use secant method to find the root of $\cos x - xe^x = 0$ correct to three decimal places.

OR

Q.3 (a) Using an indentation along a Branch cut show that $\int_0^\infty \frac{\sin x}{x}dx=\frac{\pi}{2}$. **[05]**

(b) Attempt the following. **[09]**

1. State Cauchy's residue theorem and evaluate $\int_C \frac{5z-2}{z(z-1)}dz$. Where C is the circle $|z|=2$.
2. Use power method to find the largest of Eigen values of the matrix $A=\begin{bmatrix}4 & 2\\ 1 & 3\end{bmatrix}$. Perform four iterations only.
3. Find a real root of the equation $x^3-x-1=0$ correct to four decimal places, using Newton-Raphson method.

Q.4 (a) Compute $\cosh(0.56)$ using Newton's forward difference formula and also estimate the error for the following table. **[05]**

x	0.5	0.6	0.7	0.8
$f(x)$	1.127626	1.185465	1.255169	1.337435

(b) Attempt the following. **[09]**

1. Solve the following linear system of equations by Gauss elimination method.

$$2x+y-z=1$$
$$5x+2y+2z=-4$$
$$3x+y+z=5$$

2. State Trapezoidal rule with n=10 and evaluate $\int_0^1 e^{-x^2}dx$.
3. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Gauss Quadrature of thre points. Compare the result with analytic value.

OR

Q.4 (a) State Newton's divided difference interpolation formula and compute $f(9.2)$ from the following data. **[05]**

x_j	8.0	9.0	9.5	11.0
$f(x)_j$	2.079442	2.197225	2.251292	2.397895

Q.4 (b) Attempt the following. **[09]**

1. Solve the following linear system of equations by Gauss-Seidel

$$8x + y + z = 5$$
$$x + 8y + z = 5.$$
$$x + y + 8z = 5$$

2. The speed, v meters per second, of a car, t seconds after it starts, is show in the following table.

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.08	18.90	21.60	18.54	10.26	4.50	4.5	5.4	9.0

Using Simpson's $1/3$ rule, find the distance travelled by the car in 2 minutes.

3. State Simpson's $3/8$ -rule and evaluate $\int_0^1 \frac{dx}{1+x^2}$ taking $h = \frac{1}{6}$.

Q.5 (a) Explain the Euler's method to find Numerical solution of **[05]**
$\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$.

(b) Attempt the following. **[09]**

1. Use Runge–Kutta second order method to find the approximate value of $y(0.2)$ given that $\frac{dy}{dx} = x - y^2$ & $y(0) = 1$ & $h = 0.1$

2. Determine $y(0.1)$ for the Initial value problem $\frac{dy}{dx} = x^2 + y \,\Big|\, y(0) = 1$ using Modified Euler's method.

3. Use Taylor's series method to obtain $y(1.2)$ for the initial value problem $\frac{dy}{dx} = x + y \,\Big|\, y(1) = 0$ taking h=01.

OR

Q.5 (a) Derive the Newton Raphson iterative scheme by drawing appropriate figure. **[05]**

(b) Attempt the following. **[09]**

1. Use Runge-Kutta fourth order method to find $y(1.1)$, given that $\frac{dy}{dx} = x - y,\ y(1)$=1 and $h = 0.05$.

2. Find the Lagrange interpolating polynomial from the following data

x	0	1	4	5
$f(x)$	1	3	24	39

3. Find an iterative formula for $\sqrt{N}$ and hence find $\sqrt{7}$ correct to three decimal places.