Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2015

Subject Code:2141005 **Date:**03/**06/2015**

Subject Name: Signals and Systems

Time: 10:30am-1.00pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** For each of the following systems **07**

i) $y(t) = x(t-2) + x(2-t)$

ii) $y(n) = nx(n)$

determine which of properties "memoryless", "time invariant", "linear", "casual" holds and justify your answer.

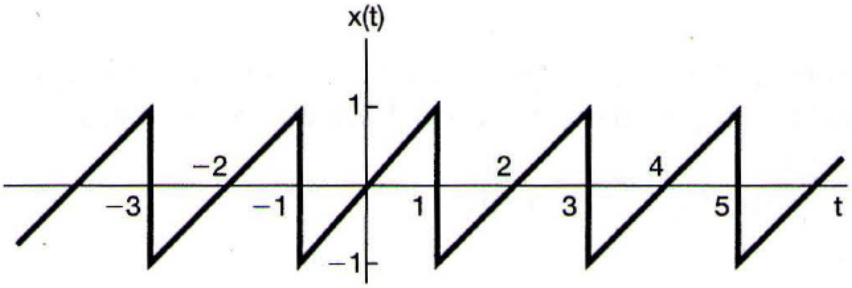
(b) Using the convolution integral to find the response *y(t)* of the LTI system with impulse response $h(t) = e^{-\beta t}u(t)$ to the input $x(t) = e^{-\alpha t}u(t)$ for $\alpha = \beta$ and $\alpha \neq \beta$. **07**

Q.2 **(a)** Determine the Fourier transform of each of the following signals: **07**

i) $x(t) = \left[e^{-\alpha t}\cos\omega_0 t\right]u(t), \quad \alpha > 0$

ii) $x[n] = \left(\frac{1}{2}\right)^{-n} u[-n-1]$

(b) Determine the Fourier series representations for the signal *x(t)* shown in figure below. **07**



OR

(b) Let *x(t)* be a periodic signal whose Fourier series coefficients are **07**

$$a_k = \begin{cases} 2, & k = 0 \\ j(\frac{1}{2})|k|, & otherwise \end{cases}$$

Use Fourier series properties to answer the following questions:

(a) Is *x(t)* real ?

(b) Is *x(t)* even ?

(c) Is $\frac{dx(t)}{dt}$ even ?

Q.3 **(a)** Consider a causal and stable LTI system S whose input x[n] and output y[n] are related through the second-order difference equation **07**

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n].$$

i) Determine the frequency response $H[e^{jw}]$ for the system S.

ii) Determine the impulse response h[n] for the system S.

(b) State and prove the following properties of the Fourier transform. **07**

i) Time Shifting

ii) Time Scaling.

OR

Q.3 (a) Determine the z-transform for the following sequences. Sketch the pole-zero plot and indicate the ROC. Indicate whether or not the Fourier transform of the sequence exists. **07**

i) $\delta[n+5]$

ii) $\left(\frac{1}{4}\right)^n u[3-n]$

(b) Determine the Laplace transform and the associated region of convergence and pole zero plot for each of the following functions of time: **07**

i) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

ii) $x(t) = \delta(t) + u(t)$

Q.4 (a) Using the long division method, determine the sequence that goes with the following *z*-transforms: $x[z] = \dfrac{1-\left(\frac{1}{2}\right)z^{-1}}{1+\left(\frac{1}{2}\right)z^{-1}}$ and x[n] is right sided. **07**

(b) Explain with example the properties and importance of LTI Systems. **07**

OR

Q.4 (a) Consider a causal LTI system whose input x[n] and output y[n] are related by the difference equation **07**

$$y[n] = \frac{1}{4}y[n-1] + x[n].$$

Determine y[n] if $x[n] = \delta[n-1]$

(b) Using the Partial fraction method, determine the sequence that goes with the following *z*-transforms: $X(z) = \dfrac{3}{z - \frac{1}{4} - \frac{1}{8}z^{-1}}$ and x[n] is absolutely summable. **07**

Q.5 (a) List the properties of the region of convergence (ROC) for the z-Transform. **07**

(b) Consider the signal **07**

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4}n\right), & n \le 0 \\ 0 & n > 0 \end{cases}$$

Determine the poles and ROC for *X[z]*.

OR

Q.5 (a) Compute and plot the convolution *y[n] = x[n]*h[n]* where **07**

$$x[n] = \begin{cases} 1, & 3 \le n \le 8 \\ 0, & otherwise \end{cases} \quad and$$

$$h[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & otherwise \end{cases}$$

(b) Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental period. **07**

i) $x(t) = [\cos(2t - \frac{\pi}{3})]^2$

ii) $x[n] = \cos(n^2 \frac{\pi}{8})$

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