Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (OLD) • EXAMINATION – WINTER 2016

Subject Code: 140001 **Date: 18/11/2016**

Subject Name: Mathematics-4

Time:02:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i) Let $z_1 = 2 + 3i$ and $z_2 = 4 - 5i$, find the value of $\frac{z_2}{z_1}$ in the form $x + iy$. **03**

(ii) Find the Principal value of $(1 + i)^{(1-i)}$. **04**

(b) (i) Let $w = f(z) = 2iz + 6\bar{z}$. Find the real and imaginary part of w and the value of $f(z)$ at $z = \frac{1}{2} + 4i$. **03**

(ii) Determine the analytic function whose real part $u = \frac{x}{x^2+y^2}$. **04**

Q.2 (a) (i) Find and sketch the sets in the complex plane given by $|z + 1| = |z - 1|$. **03**

(ii) Show that $f(z) = \begin{cases} \frac{Im\,(z^2)}{|z|} & ; z \neq 0 \\ 0 & ; z = 0 \end{cases}$ is continuous at $z = 0$. **04**

(b) (i) Evaluate $\int_C Re(z)dz$, where C the shortest path from 0 to $1 + i$. **03**

(ii) Find the bilinear transformation that maps the points $1, -1, -i$ onto $1, -1, i$. **04**

OR

(b) (i) Integrate $f(z) = Re(z)$ counterclockwise around the unit circle. **03**

(ii) Find all fixed points of $w = \frac{z+2}{z+1}$. **04**

Q.3 (a) (i) Find and sketch the image of $|z| > 1$ under the transformation $w = 3z$. **03**

(ii) Evaluate $\oint_C \frac{4-3z}{z^2-z} dz$, counterclockwise around any simple closed path such that 0 and 1 are inside C. **04**

(b) (i) Find the Maclaurin's series of the function $f(z) = e^{-2z}$. **03**

(ii) Find the Laurent series for $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center $0, |z| < 1$. **04**

OR

Q.3 (a) (i) Find and sketch the image of $x = -1$ under the transformation $w = \frac{1}{z}$. **03**

(ii) Using residue, evaluate $\oint_C \frac{1}{z^3-z^4} dz$ clockwise around the circle $C: |z| = \frac{1}{2}$. **04**

(b) (i) Expand $\frac{1}{(z+1)(z+3)}$ in Laurent series for $|z| < 1$. **03**

(ii) Find the residues at poles of $f(z) = \frac{50z}{z^3+2z^2-7z+4}$. **04**

Q.4 (a) Using Newton Raphson method find a real root of the equation $x^3 - 5x + 3 = 0$ correct to four decimal places. Take $x_0 = 2$. **07**

(b) Using Newton's divided difference formula, compute $f(0.8)$ and $f(0.9)$ from $f(0.5) = 0.479$, $f(1.0) = 0.841$, $f(2.0) = 0.909$. **07**

OR

Q.4 (a) Using Langrage interpolation formula find the value of $ln9.2$ from **07**

x	9.0	9.5	11

lnx	2.1972	2.2513	2.3979

(b) Find the positive solution of $f(x) = x - 2sinx = 0$ by secant method correct to four decimal places, start from $x_0 = 2, x_1 = 1.9$. **07**

Q.5 **(a)** Apply the Runge-Kutta's fourth order method to the initial value problem $y' = x + y; y(0) = 1$, choosing $h = 0.2$ and find $y(0.2)$. **07**

(b) Solve the following linear system of equations by Gauss sidel method. **07**
$5x + y + 2z = 19; x + 4y - 2z = -2; 2x + 3y + 8z = 39$

OR

Q.5 **(a)** Evaluate $\int_0^1 e^{-x^2} dx$ (Take $n = 10$) by **07**

(i) Trapezoidal rule

(ii) Simpson's 1/3 rule.

(b) Use Euler method to find $y(0.6)$ given that $y' = 1 - 2xy$; $y(0) = 0$, take $h = 0.2$. **07**

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