Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (New) • EXAMINATION – WINTER 2016

Subject Code: 2140001 **Date: 18/11/2016**

Subject Name:Mathematics-4

Time:02:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

MARKS

Q.1 **Short Questions** **14**

1 For the function $f(z)=\frac{|z|^2}{z}, z\neq 0$ to be continuous at $z=0$, $f(0)$ should be equal to _____.

2 $w=\log z$ is analytic everywhere except at $z=$ _____.

3 If $u+iv$ is analytic then is $v-iu$ analytic ?

4 The fixed points of the mapping $w=\frac{3iz+13}{z-3i}$ are _____.

5 The value of $\int_0^{1+i} z^2 dz$ along the line $y=x$ is _____.

6 The function $2x-x^2+py^2$ is harmonic if p equal to _____.

7 The residue of $\tan z$ at $z=\frac{\pi}{2}$ is _____.

8 The order of convergence of Newton-Raphson method is _____.

9 δ equal to _____. (In terms of E only).

10 If $y=3x^3-2x^2+1$ then $\Delta^3 y$ =_____.

11 The nth difference of a polynomial of degree n is _____.

12 Putting $n=1$ in the Newton-Cotes quadrature formula _____ rule is obtained.

13 The auxiliary quantity k_1 obtained by Runge-Kutta fourth order for the differential equation $\frac{dy}{dx}=x^2+y^2, y(0)=1$ when $h=0.1$ is _____.

14 As soon as a new value of a variable is found by iteration it is used immediately in the following equations, this method is called _____.

Q.2 **(a)** Solve the equation $z^2-(5+i)z+8i=0$. **03**

(b) Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic in some domain and find its harmonic conjugate. **04**

(c) (i) Using residue theorem, evaluate $\int_0^{2\pi}\frac{4d\theta}{5+4\sin\theta}$. **07**

(ii) Prove that $\tan^{-1} z=\frac{i}{2}\log\left(\frac{i+z}{i-z}\right)$.

OR

(c) Check whether the function defined by $f(z)=\begin{cases} u(x,y)+iv(x,y) & if\ z\neq 0 \\ 0 & if\ z=0 \end{cases}$, where $u(x,y)=\frac{x^3-y^3}{x^2+y^2}$ and **07**

$v(x,y)=\dfrac{x^3+y^3}{x^2+y^2}$ is analytic at $z=0$ or not? Also check whether Cauchy-Riemann equations are satisfied at that point?

Q.3 (a) Find and graph the strip $1<x<2$ under the mapping $w=\dfrac{1}{z}$. **03**

(b) Evaluate $\int_C \operatorname{Re}(z^2)\,dz$, where C is the boundary of the square with vertices $0,\ i,\ 1+i,\ 1$ in the clockwise direction. **04**

(c) Find the Taylor and Laurent expansion for the function $f(z)=\dfrac{z^2-1}{(z+2)(z+3)}$ in the region (i) $|z|<2$ (ii) $2<|z|<3$ (iii) $|z|>3$ **07**

OR

Q.3 (a) State Cauchy's integral formula and hence evaluate $\oint_C \dfrac{e^{2z}}{(z+1)^4}\,dz$; where C is the circle $|z|=3$. **03**

(b) Check whether the following functions are analytic or not **04**

(i) $f(z)=z^{\frac{5}{2}}$ (ii) $f(z)=\bar{z}$

(c) Define bilinear transformation. Also find the bilinear transformation which maps the points $z=1,\ i,\ -1$ into the points $w=i,\ 0,\ -i$. Hence find the image of $|z|<1$. Also draw the regions in z-plane and w-plane. **07**

Q.4 (a) State Simpson's $\dfrac{3}{8}$ rule and evaluate $\int_0^1 \dfrac{dx}{1+x^2}$ using it taking $h=\dfrac{1}{6}$. **03**

(b) Determine the interpolating polynomial of degree three using Lagrange's interpolation for the table below: **04**

x	-1	0	1	3
$f(x)$	2	1	0	-1

(c) Determine the largest eigenvalue and the corresponding eigenvector of the matrix $A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using $X^{(0)}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ **07**

OR

Q.4 (a) Solve the following linear system of equations by Gauss Elimination method **03**

$$8x_2+2x_3=-7$$
$$3x_1+5x_2+2x_3=8$$
$$6x_1+2x_2+8x_3=26$$

(b) Find the positive root of $x=\cos x$ correct to three decimal places by bisection method between 0.5 and 1. **04**

(c) Use Gauss-Seidel method to determine solution of the following system of equations using $(0,0,0)$ as initial approximation **07**

$$2x-y \quad =3$$
$$x+2y+z=3$$
$$-x \quad +z=3$$

Q.5 (a) Solve $xe^x-1=0$ correct to three decimal places between 0 and 1 using secant method. **03**

(b) Compute $\cosh(0.56)$ from the following table **04**

x	0.5	0.6	0.7	0.8
$\cosh x$	1.127626	1.185464	1.255169	1.337435

(c) Write formula of Runge-Kutta method for fourth order. Apply Euler's method to find the approximate solution of $\frac{dy}{dx} = x + y$ with $y(0) = 0$ and $h = 2$. Show your calculation up to five iterations. **07**

OR

Q.5 **(a)** The speed v meters per second of a car, t seconds after it starts is shown in the following table **03**

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.6	10.08	18.9	21.6	18.54	10.26	4.5	4.5	5.4	9.0

Using Simpson's $\frac{1}{3}$ rule, find the distance travelled by the car in 2 minutes.

(b) Develop a recurrence formula for finding $\sqrt{N}$ using Newton – Raphson method and hence compute $\sqrt{27}$ correct up to three decimal places. **04**

(c) Evaluate $\int_0^1 \frac{dt}{1+t}$ by the Gaussian formula with one point, two points and three points. **07**

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