Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(New) • EXAMINATION – WINTER 2016

Subject Code:2140105 Date:18/11/2016

Subject Name:Numerical Methods

Time:02:30 PM to 05:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1		**Short Questions**	**14**
	1	Justify whether the following Statement is true or false. ‘Runge Kutta method is better than Taylor’s method’	
	2	Write formula of Simpson’s $\frac{1}{3}$ rule for interval $[x_0, x_0 + nh]$.	
	3	Give the name of Single Step methods.	
	4	Find order of the Difference equation. $y_{n+3} - 3y_{n+1} + 2y_n = 0$	
	5	Write Iterative formula for finding the square root of N by Newton-Raphson method.	
	6	Convert equation of the curve ae^{bx} in to the linear equation.	
	7	State Gauss backward interpolation formula.	
	8	Write Laplace equation.	
	9	Write formula of first approximation for the method of false position if $f(x_0)f(x_1) < 0$	
	10	Write the difference between direct and iteration methods of solving simultaneous linear equation.	
	11	Prove that $\Delta = E\nabla = \nabla E$	
	12	Find the solution of $(E-1)^3 y_n = 1$	
	13	Determine whether the Statement is true or false “The convergence in the Gauss-Seidel method is faster than Gauss Jacobi method”	
	14	Find first approximation y_1 for IVP $\frac{dy}{dx} = x^2 + y^2, y(0) = 0$ by Picard’s method.	
Q.2	**(a)**	Perform three iterations of the bisection method to obtain the root of the equation $f(x) = x^3 - x - 1 = 0$	**03**
	(b)	Use Newton’s divided difference interpolation. Find $f(1)$ and $f(9)$ from the following table:	**04**

x	-1	0	2	5	10
y	-2	-1	7	124	999

	(c)	Find the root of the equation $\cos x = xe^x$ using Secant method correct to four decimal places.	**07**
		OR	
	(c)	Compute the real root of $x\log_{10} x - 1.2 = 0$ correct to four decimal places by false position method.	**07**
Q.3	**(a)**	Find a root of $x^4 - x^3 + 10x + 7 = 0$ using Newton-Raphson method correct up to 3 decimal places between $a = -2$ & $b = -1$	**03**
	(b)	Use Gauss elimination method to solve the equations:	**04**

$2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16$

(c) Use Gauss Seidel method to solve the following system of equation tabulating the results up to the 5th iteration. **07**

$10x - 2y + z = 12; x + 9y - z = 10; 2x - y + 11z = 20$

OR

Q.3 **(a)** By the method of least squares find the straight line that best fits the following data. **03**

x	1	2	3	4	5
y	14	27	40	55	68

(b) Determine a and b so that $y = ae^{bx}$ fit the data: **04**

x	1	2	3	4
y	7	11	17	27

(c) Solve the following system by Gauss-Jacobi method: **07**

$27x + 6y - z = 85; 6x + 15y + 2z = 72; x + y + 54z = 110$

Q.4 **(a)** Dividing he range into 10 equal parts, find the approximate value of $\int_0^{\pi} sinx dx$ by trapezoidal rule. **03**

(b) The following table gives the velocity V of a particle at time t: **04**

$t\ (seconds)$	0	2	4	6	8	10	12
$V(meter/Sec)$:	4	6	16	34	60	94	136

Find the distance moved by the particle in 12 seconds.

(c) Use Euler's method to find an approximate value of y at $x = 0.1$ in five steps. **07**

Given that $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$

OR

Q.4 **(a)** Using LaGrange's formula to find a polynomial of degree three which fits in to the data below **03**

x	-1	0	1	3
f	2	1	0	-1

(b) Use fourth order Runge- Kutta method to solve $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1$ **04**

Evaluate the value of y when $x = 0.1$

(c) Find the value of $f(2.5)$ using cubic spline interpolation for the given data. **07**

x	1	2	3	4
y	0.5	0.333	0.25	0.2

Q.5 **(a)** Prepare Newton's forward difference table for the given values of x and y. **03**

x	0	2	4	6	8	10
y	1	3	6	7	9	11

	(b)	The following IVP is given $y' = 2x + 3y,\ y(0) = 1$ Use Taylor series second order method to get $y(0.4)$ with step length $h = 0.1$	**04**
	(c)	Find the solution of the boundary value problem $y'' = y + x,\ x\epsilon\,[0,1]$ $y(0) = 0,\ y(1) = 0$ With the shooting method. Use the Runge Kutta second order to solve the initial value problem with h = 0.2.	**07**

OR

Q.5	**(a)**	Describe Galerkin method in brief.	**03**
	(b)	Prove that (1) $E = e^{hD}$ (2) $\mu = \frac{1}{2}\left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right)$	**04**
	(c)	Solve $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$ by Eulers modified method to calculate $y(0.2)$ taking $h = 0.2$	**07**

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