Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (New) • EXAMINATION – WINTER 2016

Subject Code: 2141905 **Date: 18/11/2016**

Subject Name: Complex Variables and Numerical Methods

Time: 02:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1		**Short Questions**	**14**
	1	Find the principle angle of $\frac{1}{\sqrt{3}+i}$.	
	2	Define analytic function.	
	3	Write necessary condition for differentiability of $f(z)$.	
	4	Find the real part of $f(z)=\frac{1}{z-2i}$ at point $1+i$.	
	5	Define harmonic function.	
	6	Separate real and imaginary part of $f(z)=z^2$.	
	7	Classify the singular point z=0 for the function $f(z)=\frac{1}{z^4-4z^2}$	
	8	Show that $\Delta = E-1$.	
	9	Check whether $w=\frac{1}{z}$ is conformal mapping or not.	
	10	Write trapezoidal rule.	
	11	When does Newton- Raphson method fail to find root of equation.	
	12	Can we use Gauss- Seidel method to solve the system of linear equation $3x+y+2z=8, 2x+3y+z=9, x+2y+3z=6$.	
	13	Express $y=x^2$ in factorial notation.	
	14	Find $[a,b]$ for $f(x)=\frac{1}{x}$.	
Q.2	**(a)**	Check whether functions $f(z)=z^{\frac{3}{2}}$ is analytic or not. Also find derivative of $f(z)$.	**03**
	(b)	Find the bilinear transformation which maps $z=1, i, -1$ into the points $w=0, 1, \infty$.	**04**
	(c)	Evaluate the Cauchy's principle value of $\int_{-\infty}^{\infty}\frac{dx}{(x^2+1)(x^2+9)}$.	**07**
		OR	
	(c)	Show that $u(x,y)=e^x\cos y$ is harmonic function. Also find harmonic conjugate of $u(x,y)$.	**07**
Q.3	**(a)**	Using Cauchy integral formula, Evaluate $\int_C \frac{3z^2+2}{(z-2)(z^2+4)}dz$ where C is a circle $\lvert z-2\rvert=2$.	**03**

(b) Find and plot all the roots of $\sqrt[3]{8i}$. **04**

(c) Evaluate $\int_C z^2 dz$ where C is taken along triangle in z-plane having vertices $z = \pm i, z = -1$ taken in counter clockwise sence. **07**

OR

Q.3 (a) Expend $f(z) = \dfrac{e^z}{(z-1)^2}$ about z=1. Also classify singular point z=1. **03**

(b) Discuss the continuity of $f(z) = \begin{cases} \dfrac{\bar{z}^2}{z}; & z \neq 0 \\ 0\ ; & z = 0 \end{cases}$ at z=0. **04**

(c) Expand $f(z) = \dfrac{1}{(z+2)(z+4)}$ valid for the region **07**

(i) $|z| < 2$ (ii) $2 < |z| < 4$ (iii) $|z| > 4$.

Q.4 (a) Solve by Gauss Elimination method **03**
$x + 2y + z = 3;\ 2x + 3y + 3z = 10;\ 3x - y + 2z = 13.$

(b) Derive iterative formula to find $\sqrt{N}$. Use this formula to find $\sqrt{28}$. **04**

(c) Determine the polynomial by Newton's forward difference formula from the following table: **07**

x:	0	1	2	3	4
y:	-10	-8	-8	-4	40

Also find y when x=1.5.

OR

Q.4 (a) Using Euler's method, find an approximate value of corresponding to x=0.3, given that $\dfrac{dy}{dx} = \dfrac{y-x}{y+x};\ y(0) = 1.$ Take $h = 0.1$. **03**

(b) Find a real root of the equation $x^3 - 2x - 5 = 0$ using secant method correct to three decimal places taking $x_0 = 2$ and $x_1 = 3$. **04**

(c) Determine the interpolating polynomial of degree three using Lagrange's interpolation for the table below: **07**

x:	-1	0	1	3
y:	2	1	0	-1

Also find the value of y when x=2.

Q.5 (a) The velocity v of a particle at a distant s from a point on its path is given by the following table: **03**

s (ft)	0	10	20	30	40	50	60
v(ft/sec)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft. using Simpson's $\frac{3}{8}$th rule.

(b) Using Runge- Kutta method of fourth order to calculate $y(0.2)$ given that $\dfrac{dy}{dx} = x + y;\ y(0) = 1$, taking h=0.1. **04**

(c) Solve by Gauss-Seidel method **07**
$10x + y + z = 6;\ x + 10y + z = 6;\ x + y + 10z = 6.$

OR

Q.5 (a) Evaluate $\int_0^1 \dfrac{1}{1+x} dx$ by Gaussian formula with two point. **03**

(b) Find a positive root of $xe^x - 2 = 0$ by the method of False position. **04**

(c) Find the dominant eigen value of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method and hence find the other eigen value also. **07**

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