

Seat No.: _____

Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
BE SEMESTER– 1st / 2nd (SPFU) EXAMINATION – WINTER 2016

Subject Code: MTH001

Date: 20/01/2017

Subject Name: CALCULUS

Time: 10:30 AM TO 1:30 PM

Total Marks: 70

Instruction:

1. All MCQ's are compulsory.
2. Q.1 to 25 carry 1 mark and 26 to 30 carry 2 marks.
3. Choose proper one answer, given four choices.

PART-I OBJECTIVE SECTION

Q.1 $\lim_{n \rightarrow \infty} \sum_{k=1}^{\infty} f(x_k, y_k) \delta A_k = \underline{\hspace{10cm}}$.

- A) $\int_a^b dx$, B) $\iint_R f(x, y) dA$, C) $\iiint_R f(x, y, z) dV$, D) None of these

Q.2 Double integrals gives _____.

- A) Area B) Volume C) Area & Volume D) None of these.

Q.3 $\iint_R dx dy$ means _____.

- A) Fist integrate w.r.t y then integrate w.r.t x
B) Integrate w.r.t x only
C) Integrate w.r.t y only
D) Fist integrate w.r.t y then integrate w.r.t x

Q.4 $\iint_R f(x, y) dA = \iint_{R_1} f(x, y) dA + \iint_{R_2} f(x, y) dA$ is possible if _____

- A) $R = R_1 \cup R_2$ B) $R = R_1 \cap R_2$ C) $R = R_1 \subset R_2$ D) $R = R_1 \supset R_2$

Q.5 Sum of Convergent and Divergent series are also _____

- A) Divergent B) Convergent C) Oscillating D) None of these

Q.6 If $u = x^2 + y^2$ & $y = x^3$ then $\frac{dy}{dx} = \underline{\hspace{1cm}}$.

- A) $3x^2$ B) $\frac{x}{y}$ C) $\frac{y}{x}$ D) None of these

Q.7 If $w = xy + z$ where $x = \cos t$, $y = \sin t$ & $z = t$ then $\left(\frac{dw}{dt}\right)_{t=0} = \underline{\hspace{1cm}}$.

- A) 1 B) -1 C) 2 D) -2

Q.8 $f(x, y) = \cos(2x) - x^2 e^{5y} + 3y^2$ then $f_{yx}(1, 1) = \underline{\hspace{1cm}}$

A) $-10e^5$ B) $10e^5$ C) $-5e^{10}$ D) $5e^{10}$

Q.9 At point (a, b) , $f_{yx}(a, b) = f_{xy}(a, b)$ is possible if _____

- A) $f_{yx}(a, b)$ Continuous
- B) $f_{xy}(a, b)$ Continuous
- C) $f_{yx}(a, b) \& f_{xy}(a, b)$ both continuous
- D) $f_{yx}(a, b) \& f_{xy}(a, b)$ both discontinuous

Q.10 The value of $f_{xxyz}(1,1,1)$ for $f(x, y, z) = z^3 y^2 \ln(x)$ is _____

A) -10 B) -11 C) -12 D) 12

Q.11 If $x = r \cos \theta, y = r \sin \theta$ then $r(\partial \theta / \partial x)_y =$ _____.

- A) $\sin \theta$
- B) $-\sin \theta$
- C) $-\cos \theta$
- D) $\cos \theta$

Q.12 If $\phi(x, y, z) = 0$, then $\left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y \left(\frac{\partial x}{\partial y} \right)_z =$ _____

A) -1 B) -2 C) -3 D) -4

Q.13 If $x^y = y^x$, then $\frac{dy}{dx} =$ _____ at point (1,1).

A) 1 B) -1 C) 0 D) 3

Q.14 If the partial derivative of a function of two variables exist, the function is

- A) Always continuous
- B) Always discontinuous
- C) Many not be continuous
- D) none of these

Q.15 If $x = r \cos \theta, y = r \sin \theta$ then $\frac{\partial r}{\partial x}$ is _____.

A) $\frac{r}{x}$ B) $\frac{x}{r}$ C) $-\frac{r}{x}$ D) $-\frac{x}{r}$

Q.16 If $f(x, y) = c$ then $\frac{dy}{dx}$ is _____.

B) $\frac{f_x}{f_y}$ B) $\frac{f_y}{f_x}$ C) $-\frac{f_y}{f_x}$ D) $-\frac{f_x}{f_y}$

Q.17 If $u = lx + my, v = mx - ly$ then $\left(\frac{\partial u}{\partial x} \right)_y \left(\frac{\partial x}{\partial u} \right)_v =$ _____

A) $\frac{l}{l^2 + m^2}$ B) $\frac{l^2}{l^2 + m^2}$ C) $\frac{m^2}{l^2 + m^2}$ D) $\frac{l^2 + m^2}{l^2}$

Q.18 If $x^3 + y^3 - 3axy = 0$ then $\frac{dy}{dx}$ is _____ at point (0,1)

A) a B) $-a$ C) $3a$ D) $-3a$

Q.19 The addition of two divergent series is _____

- A) Convergent
- B) Divergent
- C) Oscillating
- D) None of these

Q.20 If $u = x^2 + y^2 + z^2, x = e^{2t}, y = e^{2t} \cos 3t, z = e^{2t} \sin 3t$ then $\frac{du}{dx} =$ _____.

A) $4e^{4t}$

B) $-4e^{4t}$

C) $-8e^{4t}$

D) $8e^{4t}$

Q.21

The series $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$ is _____.

- A) Convergent B) Divergent C) finite Oscilating D) Infinitely Oscilating

Q.22

The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if _____

- A) $p < 1$ B) $p > 1$ C) $p = 1$ D) none

Q.23

For convergent of alternative series, apply following test _____

- A) Ratio test B) Root test C) Comparison test D) Leibnitz's test or Geometric series

Q.24

The series $\sum_{n=1}^{\infty} n^{-2}$ is _____

- A) Convergent B) Divergent C) finite Oscilating D) Infinitely Oscilating

Q.25

The interval of convergent of Binomial series is _____.

- A) $[-1, 1]$ B) $(-1, 1]$ C) $(-1, 1)$ D) None

Q.26

The series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is _____.

- A) Convergent B) Divergent C) finite Oscillating D) Infinitely Oscillating

Q.27

The series $\sum_{n=1}^{\infty} \sin \frac{1}{n}$ is

- A) Convergent B) Divergent C) Oscillating D) None of these

Q.28

If $u = \tan^{-1} \left(\frac{x}{y} \right)$, $x = 2t$ & $y = 1 - t^2$ then $\frac{du}{dt} =$ _____.

- A) $\frac{2}{1+t^2}$ B) $-\frac{2}{1+t^2}$ C) $-\frac{2}{1-t^2}$ D) $\frac{2}{1-t^2}$

Q.29

The value of $\int_{\frac{\pi}{2}}^{\pi} \int_0^x x \cos xy dy dx$ is

- A) 1 B) -1 C) 2 D) -2

Q.30

The value of $\int_1^2 \int_0^x \frac{dA}{x^2 + y^2}$ is

- A) $\pi \log 4$ B) $\frac{\pi}{2} \log 4$ C) $\frac{\pi}{4} \log 2$ D) $\frac{\pi}{2} \log 2$

PART-II SUBJECTIVE SECTION

Instruction:

1. Attempt any five questions out of given seven questions.
2. Each question carries seven marks.
3. Assume suitable data if necessary.

Q.1 A Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^2} x^{n-1}$ 5

B Is sequence $a_n = \frac{n}{n+1}$ monotonic increasing? 2

Q.2 A Discuss the continuity of the function $f(x, y) = \begin{cases} \frac{xy}{(x^2 + y^2)}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ 5

B Find the value of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the function $f(x, y) = x^2 + yx + y^2$ at (1,1). 2

Q.3 Evaluate: $\iint_R y^2 dA$, where R is the region bounded by $y^2 = x$ and $y = x^3$. 7

Q.4 A Test the convergence of the series $\sum_{n=1}^{\infty} \frac{4^n + 5^n}{6^n}$ 5

B Test the convergence of the series $\sum_{n=1}^{\infty} e^{-n}$. 2

Q.5 State Euler's theorem for homogeneous function. 7

If $u = \tan^{-1}(x^2 + 2y^2)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

Q.6 Evaluate $\int_0^2 \int_0^{4-x^2} \frac{x e^{2y}}{4-y} dy dx$ by changing the order of integration. 7

Q.7 Find the extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. 7
