

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**B. E. - SEMESTER -I • EXAMINATION – WINTER 2012**

Subject code: 110015

Date: 11-01-2013

Subject Name: Vector Calculus and Linear Algebra

Time: 10.30 am – 01.30 pm

Total Marks: 70

**Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1** (a) Convert the following matrix in to its equivalent Reduced Row Echelon form 04

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{bmatrix}.$$

(b) Attempt the following:

(1) Solve the system of equations by Gaussian elimination and back substitution. 03

$$x_1 + x_2 + 2x_3 = 9$$

$$2x_1 + 4x_2 - 3x_3 = 1 .$$

$$3x_1 + 6x_2 - 5x_3 = 0$$

(2) Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ . 02

(c) Attempt the following:

(1) Find inverse of matrix  $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$  using row operation. 03

$$x_1 + 2x_3 = 6 \quad 02$$

(2) Use Cramer's rule to solve  $-3x_1 + 4x_2 + 6x_3 = 30 .$

$$-x_1 - 2x_2 + 3x_3 = 8$$

**Q.2** (a) Find the rank and nullity of the matrix  $\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$ . 04

(b) Attempt the following:

(1) Find the Eigenvalues of matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 12 & 15 & 3 \end{bmatrix}$  and hence deduce 03

eigenvalues of  $A^5$  and  $A^{-1}$ .

- (2) Define Hermitian matrix and verify that the matrix 02
- $$A = \begin{bmatrix} 1 & 3+4i & -2i \\ 3-4i & 2 & 9-7i \\ 2i & 9+7i & 3 \end{bmatrix}$$
- is Hermitian matrix
- (c) Attempt the following:
- (1) Can vectors  $u_1 = 3 + x^3, u_2 = 2 - x - x^2, u_3 = x + x^2 - x^3, u_4 = x + x^2$  form 03  
basis for vector space  $P_3 = \{a + bx + cx^2 + dx^3 / a, b, c, d \in \mathbb{R}\}$  ?
- (2) Prove that the set of vectors  $A = \{(x, y) / (x, y) \in \mathbb{R}^2, x = 1\}$  can not be 02  
vector subspace of vector space  $\mathbb{R}^2$  under standard vector addition and scalar multiplication defined on  $\mathbb{R}^2$ .

- Q.3** (a) Expand the linearly independent set  $S = \{(1,1,1,1), (1,2,1,2)\}$  to be a basis for 04  
 $\mathbb{R}^4$ .
- (b) Prove that the set of all positive real numbers forms vector space under 05  
the operations defined by,  
Vector addition :  $x + y = x \cdot y$  and  
Scalar multiplication:  $\alpha \cdot x = x^\alpha$  for all  $x, y \in \mathbb{R}^+$ .
- (c) Find the co-ordinate vector of  $v = (5, -1, 9)$  relative to the basis 05  
 $S = \{v_1, v_2, v_3\}$  of  $\mathbb{R}^3$  where  $v_1 = (1, 2, 1), v_2 = (2, 9, 0)$  and  $v_3 = (3, 3, 4)$ .  
Also find the vector  $u$  in  $\mathbb{R}^3$  whose coordinate vector with respect to basis  $S$  is  $(u)_S = (-1, 3, 2)$ .

- Q.4** (a) Let  $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}, B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$  be two matrices in real matrix space 04  
 $M_{2 \times 2}$ . And let  $\langle A, B \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$ .  
Show that  $\langle A, B \rangle$  is an inner product in  $M_{2 \times 2}$ .
- (b) Find a basis for the orthogonal complement of vector subspace 05  
 $w = \text{span}\{v_1, v_2, v_3, v_4\}$  of vector space  $\mathbb{R}^5$ ,  
where  $v_1 = (2, 2, -1, 0, 1), v_2 = (-1, -1, 2, -3, 1), v_3 = (1, 1, -2, 0, -1)$   
and  $v_4 = (0, 0, 1, 1, 1)$ .
- (c) Let  $\mathbb{R}^3$  have standard Euclidean inner product. Transform the basis 05  
 $S = \{v_1, v_2, v_3\}$  in to an orthonormal basis using Gram-Schmidt process,  
where  $v_1 = (1, 0, 0), v_2 = (3, 7, -2)$  and  $v_3 = (0, 4, 1)$ .

- Q.5** (a) Is the vector  $\begin{bmatrix} 1 & 2 \\ 3 & 13 \end{bmatrix}$  in range of linear transformation  $T : P_2 \rightarrow M_{2 \times 2}$  04  
defined by,  $T(a + bt + ct^2) = \begin{bmatrix} a - 2b + c & -a + 3b \\ b + c & a + 2b + 5c \end{bmatrix}$ .
- (b) Find the associated matrix of the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ , 05  
 $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_3 + x_1)$  with the basis  
 $B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$  and  $B' = \{(1, 0, 1), (0, 1, 0), (1, 0, -1)\}$  for the domain

and co-domain respectively.

(c) Attempt the following:

- (1) Show that the Transformation,  $T : R^3 \rightarrow R^3$  defined by 03  
 $T(x_1, x_2, x_3) = (x_1 - x_2 + x_3, 3x_1 + x_2 - x_3, 2x_1 + 2x_2 + x_3)$  is linear.
- (2) Using induced matrix associated with transformation, determine the new 02  
point after applying the transformation to the given point  
 $x = (2, -6)$  rotated  $30^\circ$  in the counter clockwise direction.

**Q. 6** (a) Find the Eigenvalues and Eigenvectors of Matrix  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ . 04

(b) Find a matrix  $P$  that diagonalizes matrix  $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  and hence 05

determine  $P^{-1}AP$ .

(c) Reduce the quadratic form  $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$  in to canonical 05  
form.

**Q. 7** (a) Attempt the following:

(1) Find the directional derivative of  $f(x, y, z) = 2x^2 + 3y^2 + z^2$  at point 02  
 $P(2, 1, 3)$  in the direction of  $\vec{a} = [1, 0, -2]$ .

(2) Find divergence and curl of  $\vec{v} = xyz[x, y, z]$ . 02

(b) Verify Green's theorem for vector function 05  
 $\vec{F} = (y^2 - 7y)\hat{i} + (2xy + 2x)\hat{j}$  and curve  $C : x^2 + y^2 = 1$

(c) Evaluate  $\iint_S \vec{F} \cdot \hat{n} \, dA$  using divergence theorem. 05

Where  $\vec{F} = [\cos y, \sin x, \cos z]$  and  $S = \{(x, y, z) / x^2 + y^2 \leq 4, |z| \leq 2\}$ .

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