Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER – III • EXAMINATION – WINTER 2012

Subject code: 130001 **Date: 09-01-2013**

Subject Name: Mathematics - III

Time: 10.30 am – 01.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find a Fourier Series for f(x) = x^2, where $0 \le x \le 2\pi$ **05**

(b) $If \quad f(x) = \pi + x \ , -\pi < x < 0$ **05**

$= \pi - x \ , \quad 0 < x < \pi$

and $f(x) = f(x + 2\pi)$, $for\ all\ x$ then expand f(x) in a Fourier Series.

(c) Find a half range sine series for $f(x) = \pi - x, 0 < x < \pi$ **04**

OR

(c) Find a Fourier Series for a periodic function f(x) with a period 2,where **04**

$f(x) = -1 \quad , -1 < x < 0$

$= 1 \quad , \quad 0 < x < 1$

Q.2 **(a)** Find the inverse Laplace transforms of **05**

$(i)\dfrac{5s^2 + 3s - 16}{(s-1)(s+3)(s-2)} \quad (ii)\dfrac{s^3}{s^4 - 81}$

(b) State the Convolution theorem on Laplace transforms and using it find **05**

$L^{-1}\left[\dfrac{1}{s(s^2+4)}\right]$

(c) Find the Laplace transforms of (i) $\cos^2 2t$ and (ii) $t^3 \cosh 2t$ **04**

OR

(c) Find the Laplace transforms of the half wave rectifier **04**

$f(t) = \sin \omega t \ , 0 < t < \dfrac{\pi}{\omega}$ and $f(t) = f\left(t + \dfrac{2\pi}{\omega}\right)$

$= 0 \ , \quad \dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega}$

Q.3 **(a)** Define Beta function. **05**

Prove that (i) $\sqrt{\dfrac{1}{2}} = \sqrt{\pi}$ (ii) B(m , n) = B(m , n + 1) + B(m + 1 , n)

(b) State the Duplication formula. **05**

Show that $\int_0^\infty \sqrt{x}\, e^{-x^2}\, dx \times \int_0^\infty \dfrac{e^{-x^2}}{\sqrt{x}}\, dx = \dfrac{\pi}{2\sqrt{2}}$

(c) Show that (i) $\int_{-a}^{a} (a+x)^{m-1}(a-x)^{n-1}\, dx = (2a)^{m+n-1} B(m,n)$ **04**

(ii) $\int_0^1 \dfrac{x}{\sqrt{1-x^5}}\, dx = \dfrac{1}{5} B\left(\dfrac{2}{5}, \dfrac{1}{2}\right)$

OR

Q.3 **(a)** State the necessary and sufficient conditions to be exact differential equation. Using it, solve $x^2 y\,dx - (x^3 + y^3)dy = 0$ **05**

(b) Using the method of variation of parameters , solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \operatorname{cosec} x$ **05**

(c) Solve (i) $\frac{dy}{dx} = \cos x \cos y - \sin x \sin y$ **04**

(ii) $\frac{dy}{dx} + y \tan x = \sin 2x$

Q.4 **(a)** Using the method of undetermined coefficients , solve **05**

$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 6x + 3x^2 - 6x^3$

(b) Solve $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ **05**

(c) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x^2}$ **04**

OR

Q.4 **(a)** (i) If one of the solutions of $x^2y'' - 4xy' + 6y = 0$ *is* $y_1 = x^2, x > 0$ then determine its second solution. **05**

(ii) Solve : $y''' - y'' + 100y' - 100y = 0$

(b) Prove that, (i) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ (ii) $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$ **05**

(c) Show that (i) $J_{n-1}(x) - J_n'(x) = \frac{n}{x} J_n(x)$ (ii) $J_0(0) = 1$ **04**

Q.5 **(a)** Express $f(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. **05**

(b) Find a power series solution of $\frac{d^2y}{dx^2} + y = 0$ **05**

(c) Classify the singularities for following differential equations **04**

(i) $2x^2 \frac{d^2y}{dx^2} + 6x\frac{dy}{dx} + (x+3)y = 0$

(ii) $x(x+1)^2 y'' + (2x-1)y' + x^2 y = 0$

OR

Q.5 **(a)** Solve two dimensional Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$,using the method separation of variables. **05**

(b) A rod of length L with insulated side is initially at uniform temperature 100^0C. Its ends are suddenly cooled at 0^0C and kept at that temperature. Find the temperature $u(x, t)$. **05**

(c) Find the Fourier transform of $f(x) = xe^{-x}, x > 0$ **04**

$= 0 \quad , x < 0$

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