Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

SEMESTER– 3 EXAMINATION – WINTER 2012

Subject code: 130002 **Date: 09/01/2013**

Subject Name: Advanced Engineering Mathematics

Time:10:30 – 01:30 **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	i	Solve $\frac{dy}{dx}+\frac{1}{x^2}y=6e^{1/x}$.	**03**
		ii	Solve $x^2y\,dx-\left(x^3+xy^2\right)dy=0$.	**04**
	(b)		Find the series solution of $\left(x^2+1\right)y''+x\,y'-x\,y=0$ near $x=0$.	**07**
Q.2	**(a)**		Solve $2\frac{\partial u}{\partial x}=\frac{\partial u}{\partial t}+u$ subject to the condition $u(x,0)=4e^{-3x}$.	**07**
	(b)		Find the series solution of $y''=2y'$ in powers of x.	**07**
			OR	
	(b)	i	Solve $\frac{d^2y}{dx^2}+\frac{dy}{dx}-12y=e^{6x}$.	**03**
		ii	Solve $\left(D^2-3D+2\right)y=\frac{e^x}{1+e^x}$ by method of variation of parameters.	**04**
Q.3	**(a)**		Find the fourier series of $\begin{aligned} f(x) &= x^2 \quad (0,\pi) \\ &= 0, \quad (\pi,2\pi) \end{aligned}$.	**07**
	(b)		Find the half range cosine series for $f(x)=x,\ 0<x<3$.	**07**
			OR	
Q.3	**(a)**		Find fourier series of $f(x)=x^2+x$ where $-2<x<2$.	**07**
	(b)		Find fourier series of $\begin{aligned} f(x) &= x, \quad -1<x<0 \\ &= 2, \quad 0<x<1 \end{aligned}$.	**07**
Q.4	**(a)**		Prove that $L(1)=\frac{1}{s}$ and $L(\sinh at)=\frac{a}{s^2-a^2}$	**07**
	(b)		Evaluate	
			(i) $L^{-1}\left\{\frac{6s}{s^2-16}\right\}$	**03**
			(ii) $L^{-1}\left\{\frac{10}{(s-2)^4}\right\}$	**04**
			OR	
Q.4	**(a)**		Solve by laplace transform $y''+6y=1$, $y(0)=2$, $y'(0)=0$.	**07**
	(b)		If $L\{f(t)\}=\overline{f}(s)$ and if $L\left\{\frac{f(t)}{t}\right\}$ exists then prove that $L\left\{\frac{f(t)}{t}\right\}=\int_s^\infty \overline{f}(s)ds$. Also find $L\left\{\frac{\sin 2t}{t}\right\}$.	**07**

Q.5	**(a)**	(1)	Define following terms. (i) Error function. (ii) Sinusoidal function.	 **02** **02**
		(2)	Form partial differential equation of $z = f\left(\frac{x}{y}\right)$.	**03**
	(b)		Find the fourier integral representation of the function $f(x) = 2, \quad \lvert x \rvert < 2$ $\quad\;\; = 0, \quad \lvert x \rvert > 2$	**07**

OR

Q.5	**(a)**	(1)	Form partial differential equation of $f\left(x+y+z, x^2+y^2+z^2\right)=0$ where f is an arbitrary function.	**04**
		(2)	Solve $\frac{\partial^2 z}{\partial x^2} = z$.	**03**
	(b)	(1)	Solve $p + q^2 = 1$	**07**
		(2)	Solve $p^2 + q^2 = z^2(x+y)$	

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