

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**BE - SEMESTER-IV • EXAMINATION – WINTER 2013**

**Subject Code: 140001****Date: 17-12-2013****Subject Name: Mathematics - IV****Time: 02:30 pm to 05:30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a) (i) Write function  $f(z) = z + 1/z$  in  $f(z) = u(r, \theta) + iv(r, \theta)$  form. 08**(ii) Sketch the region  $|z - 2 + i| \leq 1$ . Is it a domain?(iii) Find the value of  $(-i)^i$ .(iv) Evaluate  $\oint_C e^{z^2} dz$  where  $C$  is any closed contour. Justify your answer.**(b) (i) Prove that  $\tan^{-1} z = \frac{i}{2} \log \left( \frac{i+z}{i-z} \right)$ . 06**(ii) Prove that  $\nabla = 1 - E^{-1}$ .(iii) Find the pole of order of the point  $z = 0$  for the function  $\frac{\sin z}{z^4}$ .**Q.2 (a) Write the Simpson's 3/8 rule for numerical integration. Using Simpson's 1/3 rule 07**evaluate  $\int_0^6 f(x) dx$  from the following data. Take  $h=1$ .

x	0	1	2	3	4	5	6
f(x)	1	0.5	0.3333	0.25	0.2	0.1666	0.1428

**(b) Explain Newton's method for solving equation  $f(x) = 0$ . Apply this method to find the iterative formula to find approximate value of  $\sqrt{N}$  and hence find  $\sqrt{7}$  correct up to three decimal. 07****OR****(b) Explain bisection method for solution of equation. Using this method find the approximate solution of  $x^3 + x - 1 = 0$  correct up to three decimal points. 07****Q.3 (a) Use Newton's forward difference method to find the approximate value of  $f(2.3)$  from the following data 07**

x	2	4	6	8
F(x)	4.2	8.2	12.2	16.2

**(b) Use Gauss Seidel method to determine roots of the following simultaneous equations. 07**

$$8x + y + z = 5$$

$$x + 8y + z = 5$$

$$x + y + 8z = 5$$

**OR****Q.3 (a) Explain quadratic Lagrange interpolation. Compute  $f(2)$  by using Lagrange interpolation method from the following data. 07**

x	-1	0	1	3
f(x)	2	1	0	-1

**(b) Write a formula for divided difference  $f[x_0, x_1]$  and  $f[x_0, x_1, x_2]$ . Using Newton's divided difference formula compute  $f(9.5)$  from the following data 07**

x	8	9	9.2	11
f(x)	2.079442	2.197225	2.219203	2.397895

- Q.4 (a)** Find the bilinear transformation that maps the points  $z_1 = 1$ ,  $z_2 = i$ ,  $z_3 = -1$  onto  $w_1 = -1$ ,  $w_2 = 0$ ,  $w_3 = 1$  respectively. Find image of  $|z| < 1$  under this transformation. **07**
- (b)** Define a harmonic function. Show that  $u(x, y) = x^2 - y^2 + x$  is harmonic. Find the corresponding analytic function  $f(z) = u(x, y) + iv(x, y)$ . **07**

**OR**

- Q.4 (a)** State de Moivre's formula. Find and graph all sixth root of unity in complex plane. **07**
- (b)** Explain translation, rotation and magnification transformation. Find the image of the  $|z - 1| = 1$  under transformation  $w = 1/z$ . **07**

- Q.5 (a)** Find Laurent's series expansion in power of  $z$  that represent  $f(z) = \frac{1}{z(z-1)}$  for domain (i)  $0 < |z| < 1$  (ii)  $0 < |z-1| < 1$  **07**

- (b)** Evaluate (i)  $\int_C z^2 dz$ , where  $C$  is line joins point  $(0,0)$  to  $(4,2)$  **07**

$$(ii) \oint_C \frac{z^2 - 4z + 4}{z + i} dz \text{ where } C \text{ is } |z| = 2$$

**OR**

- Q.5 (a)** (i) Evaluate  $\oint_C e^{\frac{3}{z}} dz$  where  $C$  is  $|z| = 1$  **07**

(ii) Evaluate  $\oint_C \frac{1}{(z-1)^2(z-3)} dz$   $C$  is  $|z| = 2$ .

- (b)** Evaluate a real integral  $\int_0^{\infty} \frac{x \sin x}{x^2 + 9} dx$  using residue. **07**

\*\*\*\*\*