

GUJARAT TECHNOLOGICAL UNIVERSITY
B. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 110009**Date: 05-01-2015****Subject Name: Mathematics - II****Time: 10:30 am - 01:30 pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Find the Rank of the matrix $\begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$ **05**
- (b) Solve the following system of equations using Gauss Elimination method **05**

$$\begin{aligned} 3x + 3y + 2z &= 1, & x + 2y &= 4 \\ 10y + 3z &= -2, & 2x - 3y - z &= 5 \end{aligned}$$
- (c) Find k , l and m so that $\begin{bmatrix} -1 & k & -i \\ 3 - 5i & 0 & m \\ l & 2 + 4i & 2 \end{bmatrix}$ is Hermitian. **04**
- Q.2** (a) Show that the set of all pairs of real numbers of the form $(1, x)$ with the operations defined as $(1, x) + (1, y) = (1, x + y)$ and $k(1, x) = (1, kx)$ is a vector space. **05**
- (b) Express the vector $(6, 11, 6)$ as a linear combination of $(2, 1, 4)$, $(1, -1, 3)$, $(3, 2, 5)$ **05**
- (c) Find the condition on a, b, c so that the vector $v = (a, b, c)$ is in the span of $\{v_1, v_2, v_3\}$ where $v_1 = (2, 1, 0)$, $v_2 = (1, -1, 2)$, $v_3 = (0, 3, -4)$ **04**
- Q.3** (a) Check whether the set $\{2 + x + x^2, x + 2x^2, 4 + x\}$ of polynomials is linearly dependent or independent in P_2 **05**
- (b) Find a basis for the subspace of P_2 spanned by the vectors $1 + x, x^2, -2 + 2x^2, -3x$ **05**
- (c) Find a basis for the row and column subspaces of $\begin{bmatrix} 1 & 4 & 5 & 4 \\ 2 & 9 & 8 & 2 \\ 2 & 9 & 9 & 7 \\ -1 & -4 & -5 & -4 \end{bmatrix}$ **04**
- Q.4** (a) Show that $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (2x - y + z, y - 4z)$ is a linear transformation. **05**
- (b) Consider the basis $S = \{v_1, v_2\}$ for R^2 where $v_1 = (-2, 1)$ and $v_2 = (1, 3)$. **05**
Let $T : R^2 \rightarrow R^3$ be the linear transformation such that $T(v_1) = (-1, 2, 0)$ and $T(v_2) = (0, -3, 5)$ then find the formula of $T(x, y)$
- (c) Let $T : R^2 \rightarrow R^2$ be the linear transformation defined by $T(x, y) = (2x - y, -8x + 4y)$ then find a basis for kernel of T and range of T **04**

- Q.5** (a) Let $T : R^3 \rightarrow R^3$ be the linear transformation defined by $T(x, y, z) = (x + 2y + z, 2x - y, 2y + z)$ then find the matrix of T with respect to the basis $\{(1, 0, 1), (0, 1, 1), (0, 0, 1)\}$ **05**
- (b) Let $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ then check whether $\langle u, v \rangle = u_1v_1 - u_2v_2 + u_3v_3$ defines an inner product on R^3 **05**
- (c) For $p = a_0 + a_1x + a_2x^2$ and $q = b_0 + b_1x + b_2x^2$ let the inner product on P_2 be defined as $\langle p, q \rangle = a_0b_0 + a_1b_1 + a_2b_2$. Let $p = 3 - x + x^2$ and $q = 2 + 5x^2$ then find $\|p\|$, $\|q\|$ and $d(p, q)$ **04**
- Q.6** (a) For $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$ let the inner product on M_{22} be defined as $\langle A, B \rangle = a_1a_2 + b_1b_2 + c_1c_2 + d_1d_2$. Let $A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$ then verify Cauchy-Schwarz inequality and find the angle between A and B **05**
- (b) Show that the set of vectors $v_1 = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$, $v_2 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$ and $v_3 = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ is orthogonal in R^3 and then convert it into an orthonormal set **05**
- (c) Find the algebraic and geometric multiplicity of each of the eigen value of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$ **04**
- Q.7** (a) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1} **05**
- (b) Find a non singular matrix which diagonalizes $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ **05**
- (c) Find the maximum and minimum values of the quadratic form $x^2 + y^2 + 4xy$ subject to the constraint $x^2 + y^2 = 1$ and also determine the values of x and y at which the maximum and minimum occur **04**
