# **GUJARAT TECHNOLOGICAL UNIVERSITY BE - SEMESTER-IV • EXAMINATION - WINTER • 2014**

## Subject Code: 140001 **Subject Name: Mathematics - IV**

# Date: 19-12-2014

## Time: 02:30 pm - 05:30 pm

## **Total Marks: 70**

04

**Instructions:** 

- 1. Attempt all questions.
- Make suitable assumptions wherever necessary. 2.
- 3. Figures to the right indicate full marks.

#### Find the principal argument Arg z, when $z = \frac{-2}{1+\sqrt{3}i}$ . Q.1 **(a)** 03 (i) (ii) Sketch the following sets and determine which are domains :

- (a) Im z > 1(b)  $0 \le \arg z \le \frac{\pi}{4}$
- (b) (i) Using the definition of limit, show that if f(z) = iz in the open disk 04 |z| < 1, then  $\lim_{z \to 1} f(z) = i$ 
  - 03 (ii) Show that  $f(z) = |z|^2$  is continuous at each point in the plane, but not differentiable.

(a) (i) Define the singular point of the function and state where the function 0.2 03  $f(z) = \frac{z^2+1}{(z+2)(z^2+2z+2)}$  is analytic?

(ii) show that 
$$\overline{\sin(iz)} = \sin(i\overline{z})$$
 if and only if  $z = n\pi i$   $(n \in \mathbb{Z})$  04

- (b) (i) If C is any simple closed contour, in either direction, then show that 03  $\int_C \exp(z^3) \, dz = 0$ 
  - (ii) Find the value of integral  $\int_c \bar{z} \, dz$  where c is the right-hand half 04  $z = 2 e^{i\theta} \left(\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}\right)$  of the circle |z| = 2, from z = -2i to z = 2i.

#### OR

**(b)** (i) If 
$$f(z)$$
 is analytic function, prove that  $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log |f'(z)| = 0.$  04

(ii)Examine the analyticity of sinh z. 03

(a) Define residue at simple pole and find the residues of the function **Q.3**  $f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2+4)}$  at each of its poles in the finite z-plane. 07

#### (b) State and Prove Cauchy's Integral formula. 07 OR

Q.3 (a) Expand 
$$f(z) = \frac{-1}{(z-1)(z-2)}$$
 in the regions (a)  $|z| < 1$  (b)  $1 < |z| < 2$   
(c)  $|z| > 2$  07

(b) Write Stirling's interpolation formula and find the value of f(2.73), using Bessel's interpolation formula up to four differences, from the following table

|      | -      |        | -      |        |        |        |
|------|--------|--------|--------|--------|--------|--------|
| x    | 2.5    | 2.6    | 2.7    | 2.8    | 2.9    | 3.0    |
| f(x) | 0.4938 | 0.4953 | 0.4965 | 0.4974 | 0.4981 | 0.4987 |

(a) 1. Evaluate  $\int_0^1 \exp(-x^2) dx$  by using the Gaussian integration formula for n = 3. 03 Q.4 2. Solve the following system of equations using Gauss-Seidel method correct up to three decimal places.

$$60x - 4y + 6z = 150, 2x + 2y + 18z = 30, x + 17y - 2z = 48$$

(b) State Trapezoidal rule with n = 10 and using it, evaluate  $\int_0^1 2e^x dx$ .

OR

(a) From the following table, find P when  $t = 142^{\circ}C$  and  $175^{\circ}C$ , using appropriate **Q.4** 07 Newton's interpolation formula.

| Temp. $t^0 C$ | 140  | 150  | 160  | 170  | 180   |
|---------------|------|------|------|------|-------|
| Pressure P    | 3685 | 4854 | 6302 | 8076 | 10225 |

- (b) Derive an iterative formula to find  $\sqrt{N}$  and hence find approximate value of 07  $\sqrt{65}$  and  $\sqrt{3}$ , correct up to three decimal places.
- Q.5 (a) Find numerically smallest Eigen value of the given matrix using power method, 07 correct up to three decimal places.

| [-15 | 4   | 3] |
|------|-----|----|
| 10   | -12 | 6  |
| 20   | -4  | 2  |

(b) Apply improved Euler method to solve the initial value problem y' = x + y07 with y(0) = 0 choosing h = 0.2 and compute  $y_1, y_2, y_3, y_4, y_5$ . Compare your results with the exact solutions.

OR

(a) y(12) by Lagrange Interpolation formula from following values. Q.5

| x | 11 | 13 | 14 | 18 | 20  | 23  |
|---|----|----|----|----|-----|-----|
| у | 25 | 47 | 68 | 82 | 102 | 124 |

(**b**) Prove the following

| (i) $\Delta = E - 1$       | 02 |
|----------------------------|----|
| (ii) $\nabla = 1 - E^{-1}$ | 02 |

(iii)  $hD = log (1 + \Delta)$ 03

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