

Seat No.: _____

Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
DIPLOMA ENGINEERING – SEMESTER – I/II • EXAMINATION – WINTER- 2016

Subject Code: 3326302

Date: 04.01.2017

Subject Name: Engineering Maths II

Time: 02:30 PM TO 05:00 PM

Total Marks: 70

Instructions:

1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible. (Scientific/Higher Version not allowed)
5. English version is authentic.

Q.1 Fill in the blanks using appropriate choice from the given options. 14

- 1 If $v = x^4 + 5xy + 7$ then $\frac{\partial v}{\partial y} =$ _____
a. $4x^2 + x$ b. $4x^3$ c. $5x$ d. $x^4 + 5x$
- 2 $\nabla v = x^4 + 5xy + 7$ તો $\frac{\partial v}{\partial y} =$ _____
અ. $4x^2 + x$ અ. $4x^3$ સ. $5x$ સ. $x^4 + 5x$
- 2 If $f = x^3y^3z^3$ then $\frac{\partial f}{\partial x} =$ _____
a. $3x^2y^3z^3$ b. $3x^3y^3z^3$ c. $3xyz$ d. $3x^2y^2z^2$
- 3 $\nabla f = x^3y^3z^3$ તો $\frac{\partial f}{\partial x} =$ _____
અ. $3x^2y^3z^3$ અ. $3x^3y^3z^3$ સ. $3xyz$ સ. $3x^2y^2z^2$
- 3 $\frac{d}{dx}(5x^2 + 7x - 6) =$ _____
a. $10x+7$ b. $10x-7$ c. $5x+7$ d. $5x-7$
- 3 $\frac{d}{dx}(5x^2 + 7x - 6) =$ _____
અ. $10x+7$ અ. $10x-7$ સ. $5x+7$ સ. $5x-7$
- 4 If $f(x) = \sin x$ then $f'(\frac{\pi}{3}) =$ _____
a. $-\frac{1}{2}$ b. $\frac{\sqrt{3}}{2}$ c. $\frac{1}{2}$ d. 2
અ. $-\frac{1}{2}$ અ. $\frac{\sqrt{3}}{2}$ સ. $\frac{1}{2}$ સ. 2
- 4 $\nabla f(x) = \sin x$ તો $f'(\frac{\pi}{3}) =$ _____
અ. $-\frac{1}{2}$ અ. $\frac{\sqrt{3}}{2}$ સ. $\frac{1}{2}$ સ. 2
- 5 Complementary function of $(4D^2 - 4D + 1) = 4$ is _____
a. 0 b. $(c_1 + c_2x)e^x$ c. $(c_1 + c_2x)e^{\frac{x}{2}}$ d. 1
- 5 $(4D^2 - 4D + 1) = 4$ નું પુરક કાર્ય વિધેય = _____
અ. 0 અ. $(c_1 + c_2x)e^x$ સ. $(c_1 + c_2x)e^{\frac{x}{2}}$ સ. 1

- 6** If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then the differential equation is _____
 a. Non Exact b. Bernoulli c. Exact d. None of these
- 5** જો $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ હોય તો માટે નું વિકલ સમીકરણ = _____
 અ. Non Exact બિ. Bernoulli ચિ. Exact સ. None of these
- 7** $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} =$ _____
 a. -1 b. 1 c. 0 d. ∞
- 9** $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} =$ _____
 a. -1 b. 1 c. 0 d. ∞
- 8** $\Gamma 6 =$ _____
 a. $7!$ b. $8!$ c. $6!$ d. $5!$
- 6** $\Gamma 6 =$ _____
 અ. $7!$ બિ. $8!$ ચિ. $6!$ સ. $5!$
- 9** If each of r, θ is a function of the variable x, y then the jacobian $\frac{\partial(r, \theta)}{\partial(x, y)}$ is determinant of order _____
 a. 2×2 b. 3×3 c. 1×1 d. $n \times n$
- 6** જો r, θ એ x, y માં વિધેય હોય તો jacobian $\frac{\partial(r, \theta)}{\partial(x, y)}$ ની નિશ્ચાયક નો કરું _____
 અ. 2×2 બિ. 3×3 ચિ. 1×1 સ. $n \times n$
- 10** If $x = r \cos \theta, y = r \sin \theta$ then $\frac{\partial(r, \theta)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(r, \theta)} =$ _____
 a. 0 b. 1 c. 2 d. ∞
- 10** જો $x = r \cos \theta, y = r \sin \theta$ ત્થા $\frac{\partial(r, \theta)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(r, \theta)} =$ _____
 અ. 0 બિ. 1 ચિ. 2 સ. ∞
- 11** $\int x^7 dx =$ _____ +c
 અ. $\frac{x^7}{7}$ બિ. x^7 ચિ. x^8 સ. $\frac{x^8}{8}$
- 11** $\int x^7 dx =$ _____ +c
 અ. $\frac{x^7}{7}$ બિ. x^7 ચિ. x^8 સ. $\frac{x^8}{8}$
- 12** Wronskian of y_1 and y_2 is _____
 a. $\begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix}$ b. $\begin{vmatrix} 1 & 1 \\ y_1 & y_2 \end{vmatrix}$ c. $\begin{vmatrix} y'_1 & y'_2 \\ y_1 & y_2 \end{vmatrix}$ d. $\begin{vmatrix} y_1 & y'_1 \\ y_2 & y'_2 \end{vmatrix}$
- 12** y_1 અને y_2 નું Wronskian = _____
 અ. $\begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix}$ બિ. $\begin{vmatrix} 1 & 1 \\ y_1 & y_2 \end{vmatrix}$ ચિ. $\begin{vmatrix} y'_1 & y'_2 \\ y_1 & y_2 \end{vmatrix}$ સ. $\begin{vmatrix} y_1 & y'_1 \\ y_2 & y'_2 \end{vmatrix}$
- 13** Integrating factor of $\frac{dy}{dx} + \frac{y}{x} = e^x$
 a. X^2 b. X c. $\log x$ d. None of these
- 13** $\frac{dy}{dx} + \frac{y}{x} = e^x$ નું સંકલયકારક અવયવ = _____
 અ. X^2 બિ. X ચિ. $\log x$ સ. None of these

- 14 If $v = x^3 + 3y + xy$ then $\frac{\partial v}{\partial y} = \underline{\hspace{2cm}}$
 a. $3+x$ b. $3x^2+y$ c. $3-y$ d. $3y+x$
- 18 જે $v = x^3 + 3y + xy$ તો $\frac{\partial v}{\partial y} = \underline{\hspace{2cm}}$
 અ. $3+x$ બ. $3x^2+y$ ચ. $3-y$ સ. $3y+x$

Q.2 (a) Attempt any two કોઇપણ બે ના જવાબ આપો.

06

1. Evaluate : $\lim_{x \rightarrow 0} \left[\frac{1}{x} - \frac{1}{e^x - 1} \right]$
2. કિમત શોધો : $\lim_{x \rightarrow 0} \left[\frac{1}{x} - \frac{1}{e^x - 1} \right]$
3. Find the value of $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ at the point $(2,1,1)$ for $f(x,y,z) = 2x^2yz - x^4z^3 + 2y^4$
4. $f(x,y,z) = 2x^2yz - x^4z^3 + 2y^4$ માટે બિંદુ $(2,1,1)$ પાસે $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ ની કિમત શોધો.
5. Find Maclaurin's series of $\cos x$ up to 5th power.
6. $\cos x$ ની 5th કર્મ માટે Maclaurin's series શોધો.

(b) Attempt any two કોઇપણ બે ના જવાબ આપો.

08

1. If $x = r\sin\theta\cos\varphi, y = r\sin\theta\sin\varphi$ and $z = r\cos\theta$ then show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2 \sin\theta$
2. જે $x = r\sin\theta\cos\varphi, y = r\sin\theta\sin\varphi$ and $z = r\cos\theta$ તો સાબિત કરો $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2 \sin\theta$
3. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$
4. જે $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ હોય, તો દશોવો કે $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$
5. Evaluate : $\int_0^7 \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}} dx$
6. કિમત શોધો : $\int_0^7 \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}} dx$

Q.3 (a) Attempt any two કોઇપણ બે ના જવાબ આપો.

06

1. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$
2. કિમત શોધો : $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$
3. Prove that : $\Gamma n + 1 = n!$
4. સાબિત કરો : $\Gamma n + 1 = n!$
5. Evaluate: $\int_0^\infty e^{-\sqrt{x}} \sqrt{x} dx$
6. કિમત શોધો: $\int_0^\infty e^{-\sqrt{x}} \sqrt{x} dx$

(b) Attempt any two કોઇપણ બે ના જવાબ આપો.

08

1. Simplify : $B(m,n) \cdot B(m+n,p) \cdot B(m+n+p,q)$
2. સાદુ રૂપ આપો: $B(m,n) \cdot B(m+n,p) \cdot B(m+n+p,q)$
3. Find maximum and minimum value of the function $f(x) = 2x^3 - 15x^2 + 36x + 10$

2. $f(x) = 2x^3 - 15x^2 + 36x + 10$ માટે મહત્વમાને નુંન્યતમ કિમત મેળવો.
3. Find the surface area of that portion of the sphere $x^2+y^2+z^2 = a^2$ that is above the xy -plane and within the cylinder $x^2+y^2 = b^2$ $0 < b < a$
3. નંદિકાર $x^2 + y^2 = b^2$, $0 < b < a$. ની અંદર xy સમતલ ની ઉપર આવેલા ગોલક $x^2 + y^2 + z^2 = a^2$ ની heat portion નું ક્ષેત્રફળ શોધો.

Q.4	(a)	Attempt any two કોઇપણ બે ના જવાબ આપો.	06
1.	Solve $(x^3+3xy^2) dx + (3x^2y+y^3)dy = 0$		
1.	ઉકેલો $(x^3+3xy^2) dx + (3x^2y+y^3)dy = 0$		
2.	Solve Bernoulli's differential equation $\frac{dy}{dx} = x^3y^3 - xy$		
2.	Bernoulli નું વિકલ સમિકરણ ઉકેલો $\frac{dy}{dx} = x^3y^3 - xy$		
3.	Solve $(D^2+5D+6) = e^x$		
3.	ઉકેલો $(D^2+5D+6) = e^x$		
(b)	Attempt any two કોઇપણ બે ના જવાબ આપો.		08
1.	Solve by exact differential equation $(x^2+y^2+1)dx - 2xydy = 0$		
1.	ઉકેલો $(x^2+y^2+1)dx - 2xydy = 0$		
2.	Solve $(D^2+a^2)y = \sec ax$		
2.	ઉકેલો $(D^2+a^2)y = \sec ax$		
3.	Solve Cauchy linear equation $x^2y'' - 20y = 0$		
3.	cauchy's linear સમિકરણ ઉકેલો $x^2y'' - 20y = 0$		
Q.5	(a)	Attempt any two કોઇપણ બે ના જવાબ આપો.	06
1.	If $u = x^2zi - 2y^3z^2j + xy^2zk$ then find divergence at the point $(1, -1, 1)$		
1.	જો $u = x^2zi - 2y^3z^2j + xy^2zk$ હોય તો બિંદુ $(1, -1, 1)$ પાસે divergence શોધો		
2.	Evaluate $\int_C 2x \, dx$ where $C = C_1 \cup C_2$, C_1 being the arc of the parabola $y = x^2$ between $(0,0)$ and $(1,1)$ and C_2 being the vertical line from $(1,1)$ to $(1,2)$		
2.	કિમત શોધો: $\int_C 2x \, dx$ જ્યાં $C = C_1 \cup C_2$, C_1 being the arc of the parabola $y = x^2$ between $(0,0)$ and $(1,1)$ and C_2 being the vertical line from $(1,1)$ to $(1,2)$		
3.	If $u = 7x^2y^2zi - 4y^3j + 6y^2z^3k$ then find divergence at the point $(-2, 1, 1)$		
3.	જો $u = 7x^2y^2zi - 4y^3j + 6y^2z^3k$ હોય તો બિંદુ $(-2, 1, 1)$ પાસે divergence શોધો		

(b) Attempt any two કોઇપણ બે ના જવાબ આપો.

08

1. If $\mathbf{F} = 4x^3yz^2\mathbf{i} - 12xyz^2\mathbf{j} + 13x^3y^3z\mathbf{k}$ then find curl of \mathbf{F} at point (1,2,1)
જો $\mathbf{F} = 4x^3yz^2\mathbf{i} - 12xyz^2\mathbf{j} + 13x^3y^3z\mathbf{k}$ હોય તો $\nabla \times (\mathbf{F})$ (1,2,1) માટે curl શોધો
2. If $\mathbf{F} = (xyz - y^2z + 4yz)\mathbf{i} + (xy^2 + 2xz - 4yx)\mathbf{j} + (2xy + 4yz^2)\mathbf{k}$ then find curl of \mathbf{F} at point (2,1,2)
જો $\mathbf{F} = (xyz - y^2z + 4yz)\mathbf{i} + (xy^2 + 2xz - 4yx)\mathbf{j} + (2xy + 4yz^2)\mathbf{k}$ હોય તો $\nabla \times (\mathbf{F})$ (2,1,2) માટે curl શોધો
3. If $f(x,y,z) = x^2z + 2xyz + 7x^2y$ then find grad f at the point (-2,1,0)
જો $f(x,y,z) = x^2z + 2xyz + 7x^2y$ હોય તો ∇f (-2,1,0) પાસે gradient શોધો
