

GUJARAT TECHNOLOGICAL UNIVERSITY**MCA Sem-I Remedial Examination April 2010****Subject code: 610003****Subject Name: Discrete Mathematics For Computer Science****Date: 06 / 04 / 2010****Time: 12.00 noon – 02.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Answer the following:

- (i) Express the following using predicates, quantifiers, and logical connectives. Also verify the validity of the consequence. **04**

Everyone who graduates gets a job.

Ram is graduated.

Therefore, Ram got a job.

- (ii) Prove by contradiction that $\sqrt{2}$ is an irrational number. **03**

(b) Draw Hasse Diagram of the poset $\langle \{2, 3, 5, 6, 9, 15, 24, 45\}, D \rangle$. Find **07**

- (i) Maximal and Minimal elements
- (ii) Greatest and Least members, if exist.
- (iii) Upper bound of $\{9, 15\}$ and l.u.b. of $\{9, 15\}$, if exist.
- (iv) Lower bound of $\{15, 24\}$ and g.l.b. of $\{15, 24\}$, if exist.

Q.2 (a) When a poset said to be lattice? Explain. Is every poset a lattice? Justify. **07**

Is the poset $\langle \{\emptyset, \{p\}, \{q\}, \{p, q, r\}\}, \subseteq \rangle$ lattice?

(b) Show that the lattice $\langle S_n, D \rangle$ for $n = 100$ is isomorphic to the direct product of lattices for $n = 4$ and $n = 25$. **07**

OR

(b) With proper justification give an example of **07**

- (1) A bounded lattice which is complemented but not distributive.
- (2) A bounded lattice which is distributive but not complemented.
- (3) A bounded lattice which is neither distributive nor complemented.
- (4) A bounded lattice which is both distributive and complemented.

Q.3 (a) Answer the following:

- (i) Define sub-Boolean algebra. State the necessary and sufficient condition for a subset becomes sub-algebra. Find all sub Boolean algebra of $\langle S_{110}, D \rangle$. **05**

- (ii) Prove the following Boolean identities: **02**

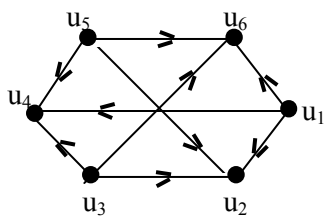
$$(1) (x' \oplus y) * (x \oplus y) = y \quad (2) (x \oplus y \oplus z) * (y \oplus z) = (y \oplus z)$$

(b) Use the Quine-Mccluskey algorithm to find the prime implicants and also obtain a minimal expression for function: $f(a, b, c, d) = \sum (15, 14, 13, 6, 5, 2, 1)$. **07**

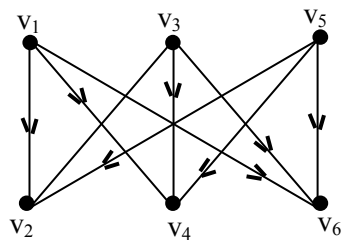
OR

Q.3 (a) Use Karnaugh map to find a minimal sum-of-product expression for the function given by $\sum (0, 1, 2, 3, 6, 7, 13, 14)$ in four variables w, x, y, z. **07**

- (b) Answer the following:
- (i) Given an expression $\alpha(a, b, c, d) = \sum (2, 3, 6, 8, 12, 15)$, determine the value of $\alpha(3, 5, 10, 30)$ where $3, 5, 10, 30 \in \langle S_{30}, D \rangle$. **04**
- (ii) Find the sum of products expansions of Boolean functions **03**
 $f(x, y, z) = (x + z)y$
- Q.4** (a) Define group homomorphism; prove that group homomorphism preserves identities, inverses and subgroups. **07**
- (b) Define cyclic group. Find generators of $\langle Z_{12}, +_{12} \rangle$. Also find its all subgroups. Which subgroups are isomorphic to $\langle Z_4, +_4 \rangle$? Justify. **07**
- OR**
- Q.4** (a) Show that if every element in a group is its own inverse, then the group must be abelian. Is the converse true? Justify. **07**
- (b) Define symmetric group $\langle S_3, \diamond \rangle$. Write its composition table. Determine all the proper subgroups of $\langle S_3, \diamond \rangle$. Which subgroup is normal subgroup? Support your answer with reason. **07**
- Q.5** (a) Define isomorphic graphs. Determine whether the digraphs G and H given in figure – 1 (a), (b) are isomorphic. **07**
- (b) Define node base of a digraph. Find all node base of the digraph shown in figure – 2. List out all the properties of a node base. Explain why no node in node base is reachable from any other node in node base. **07**
- OR**
- Q.5** (a) Define: path, simple path, elementary path. For the graph given in Figure – 3: **07**
- (i) Find an elementary path of length 2 from v_1 to v_3 .
- (ii) Find a simple path from v_1 to v_3 , which is not elementary.
- (iii) Find all possible paths from node v_2 to v_4 and how many of them are simple and elementary?
- (b) Define a directed tree. Draw the graph of the tree represented by **07**
 $(A(B(C(D)(E))(F(G)(H)(J)))(K(L)(M)(N(P)(Q(R))))$ Obtain the binary tree corresponding to it.



Graph G:
Figure – 1 (a)



Graph H:
Figure – 1 (b)

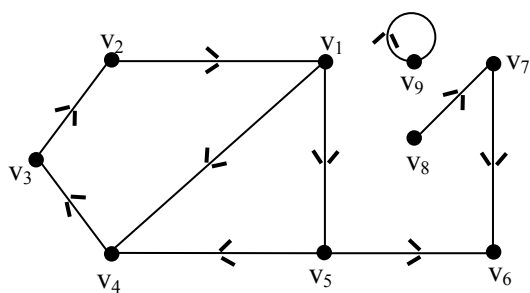


Figure – 2

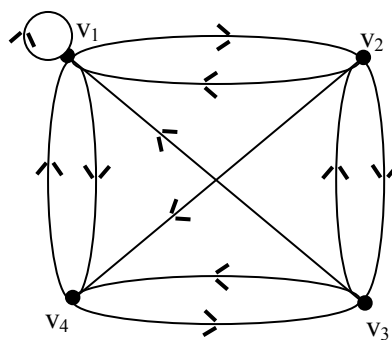


Figure – 3