Seat No.: _____ Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**MCA Integrated - SEMESTER–II • EXAMINATION – SUMMER • 2014**

**Subject Code: 4420601** **Date: 18-06-2014**

**Subject Name: Discrete Mathematics for Computer Science**

**Time: 10:30 am - 01:00 pm** **Total Marks: 70**

**Instructions:**

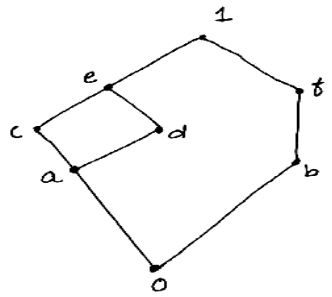
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** With proper justification give an example of **07**

i) A bounded lattice which is complemented but not distributive.
ii) A bounded lattice which is distributive but not complemented.
iii) A bounded lattice which is neither distributive nor complemented.
iv) A bounded lattice which is both distributive and complemented.

**(b)** When a poset is said to be a lattice? Explain. Is every poset a lattice? Justify. Is the poset {Φ, {p}, {q}, {p,q,r}, ⊆} a lattice? **07**

**Q.2** **(a)** Define: Chain. Determine join-irreducible elements, meet-irreducible elements, atoms and anti-atoms for the lattices shown in the Figure below: **07**




**(b)** Define: Boolean Algebra. Find all Sub Boolean algebra of Boolean algebra ‹ $S_{30}$, Λ, V, ‘ , 0 , 1 ›. **07**

**OR**

**(b)** i) Given an expression α (a,b,c,d) = ∑ (2,3,6,8,12,15), determine the value of α (3,5,10,30) where 3,5,10,30 Ɛ <$S_{30}$, D> **05**

ii) Show that **02**

a) a + a’ = 1
b) a + 0 = a

where a + b = (a * b’) join (a’ * b)

**Q.3** **(a)** Use the K-map representation to find a minimal sum-of-products expression for the following function: **07**

a) *f* (a,b,c,d) = ∑ (0,5,7,8,12,14)
b) *f* (a,b,c,d) = ∑ (5,7,10,13,15)

**(b)** Use the Quine-McCluskey representation to find a minimal SOP expression: *f* (a,b,c,d) = ∑ (0,1,6,7,8,913,14,15) **07**

**OR**

**Q.3** **(a)** i) Let(B, *, ’, 0, 1) be a boolean algebra in any Boolean algebra prove the following: a = b ⇔ ab’ + a’b = 0. **03**

ii) Find the sum of products expression of Boolean function. **03**
f(x,y,z) = (x + z) * y

iii) Define: Sub-Lattice. **01**

**(b)** Show that the lattice $<S_n, D>$ for n = 216 is isomorphic to the direct product of lattices for n = 8 and n = 27. **07**

**Q.4** **(a)** i) Define an abelian group. Show that if every element in a group is its own inverse, then the group must be abelian. **03**

ii) Show that every subgroup of a cyclic group is normal. **03**

iii) Let (G, *) be a group. Let G = 5 . How many subgroups are there of G? Why ? **01**

**(b)** Show that the set of all positive rational number forms an abelian group under the composition defined by a * b = ab/2 . **07**

**OR**

**Q.4** **(a)** Define: Group & Cyclic Group. Find the generator of ( $Z_5$* , *5 ). **07**

**(b)** Find in-degree and out-degree of each node from the following adjacency matrix A and draw its diagraph. **07**

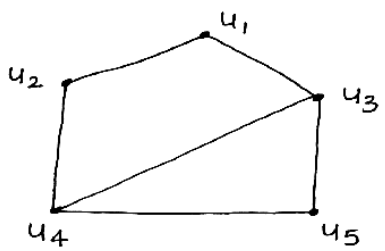
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

**Q.5** **(a)** Define: Binary tree, Loop, Null graph. Show through two examples with $n_t = 7$ and $n_t = 8$ of complete binary trees that the total number of edges is given by $2(n_t - 1)$, where $n_t$ is the number of terminal nodes. **07**

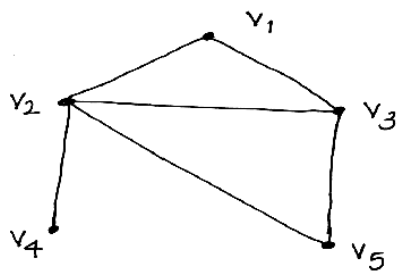
**(b)** Define Directed tree. Give three different tree representations of the following: **07**
(v0(v1(v2)(v3)(v4))(v5(v6)(v7)(v8)(v9))(v10(v11)(v12)))

**OR**

**Q.5** **(a)** Define: Isomorphic Graph, Edge Simple. Verify that, are the following graphs are isomorphic? **07**

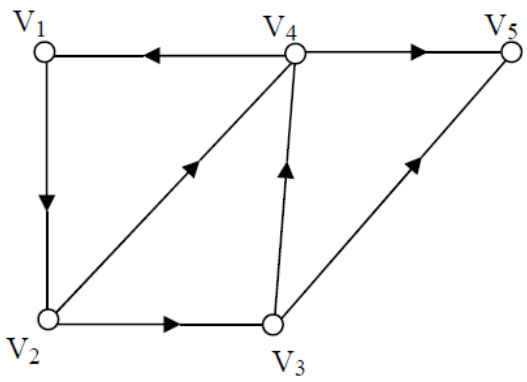



Graph – 1




Graph - 2

**(b)** Define: Node Base of a simple diagraph. Find the reachability set of all nodes for the digraph given in figure given below: Also find the nodebase for it. Is the graph Strongly or unilaterally connected? **07**

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