Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER–I • EXAMINATION – SUMMER • 2015

Subject Code: 2610003 Date: 07-05-2015

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 10:30 am to 01:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define distributive lattice. Prove that every chain is distributive lattice. Prove that the power set lattice is a complemented distributive lattice. **07**

(b) 1) Give an example of a relation which is neither reflexive nor irreflexive. **02**

2) Give an example of a relation which is both symmetric and antisymmetric **02**

3) Given S = {1,2,é é ..,10} and a relation R on S where **03**
R = {<x, y> / x + y = 10}
what are the properties of the relation R?

Q.2 **(a)** 1) Determine the truth value of each statement given below. The domain of discourse is the set of real numbers. Justify your answers. **03**

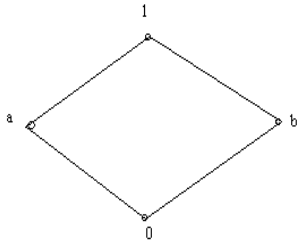
i) For every x, $x^2 > x$

ii) For some x, $x^2 > x$

iii) For every x, if x > 1 then $x^2 > x$.

2) Prove using Indirect proof technique that if n2 + 3 is odd then n is even. **04**

(b) 1) Given an expression Ŭ(x1, x2, x3) defined to be × (0, 3, 5, 7), determine the value of Ŭ(a, b, 1), where a, b, 1 ∈ B and <B, *, ⊕, ó, 0, 1> is a Boolean algebra given in the following figure. **04**



2) Define poset, lattice and chain. Give one example of each with your justification. Draw their Hasse diagrams also. **03**

OR

(b) 1) Draw the Hasse diagram of the following lattices. <S_{100}, D>, <S_{36}, D>, <S_4 x S_{25}, D>, <S_6 x S_6, D>. Which of them are isomorphic? **04**

2) Define complemented lattice. Find the compliments of every elements of the lattice <Sn, D> for n = 75. **03**

Q.3 **(a)** Define Boolean function. Use the Karnaugh map representation to find a minimal sum of products expression of the following function : **07**
f(x1, x2, x3, x4) = × (0, 1, 2, 3, 6, 7, 13, 14)

(b) 1) Let (B, ∗, ⊕, ô, 0, 1) be a Boolean algebra prove the following : **04**
a = b ⇔ (a ∗ bô) ⊕ (aô ∗ b) = 0

2) Obtain the sum of product canonical form of Boolean expression in three variables x1, x2, x3 for (x1 ⊕ x2) ∗ x3 **03**

OR

Q.3 **(a)** Define ñUniversal quantifierò and ñExistential quantifierò. **07**

(1) Formulate the symbolic expression for

i) p Ÿ q

ii) (p ∨ q)ô↔ r

in words using:

p : Today is Monday

q : It is raining

r : It is hot.

(2) State the rule UG in the predicate calculus. Verify whether the following conclusion is valid or not.

(x)(P(x)Ÿ Q(x)), ⌉Q(a) => (x)⌉P(x)

(b) Use the Quine-McClusky algorithm to obtain the minimal SOP form of the function f(a, b, c, d) = × (0, 2, 5, 7, 8, 10, 13, 15) **07**

Q.4 **(a)** Define cyclic group. Show that cyclic group is abelian but converse is not true. Is $<z_5, +_5>$ a cyclic group? If so, find its generators. **07**

(b) Define symmetric group $<S_3, \Diamond>$. Write all its elements and composition table. Show that it is non-abelian. Determine all the proper subgroups of $<S_3, \Diamond>$. **07**

OR

Q.4 **(a)** Define left coset of a subgroup <H, *> in the group <G, *>. Find left cosets of {[0], [3]} in the group $<Z_6, +_6>$. **07**

(b) Define kernel of a group homomorphism. If <G, *> and <H, æ> are two group and g: G → H is a homomorphism, show that ker(g) is a normal subgroup of <G, *>. **07**

Q.5 **(a)** Give an abstract definition of graph. When are two simple graphs said to be isomorphic? Give an example of two simple digraphs having 4 nodes and 4 edges which are not isomorphic. **07**

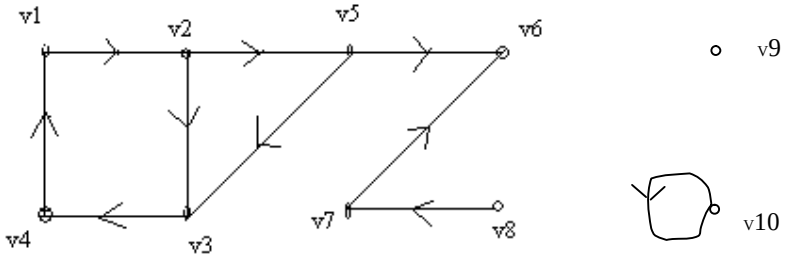
(b) Define a directed tree. Draw the graph of the tree represented by **07**
(A(B(E(H)(I))(F(J)(K))(G(L)))(C(M(O))(N(P)(Q)))(D(R(S(V))(T)(U)))
Obtain the binary tree corresponding to it.

OR

Q.5 **(a)** 1) Define Cyclic graph, Null graph, and Strongly connected graph. **03**

2) Define Adjacency matrix and path matrix of a graph. Explain each with example. **04**

(b) Define nodebase of a simple diagraph. Find the reachability set of all nodes for the diagraph given in following figure. Also find the nodebase for it. **07**



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