## **GUJARAT TECHNOLOGICAL UNIVERSITY** MCA Integrated - SEMESTER-II • EXAMINATION – SUMMER • 2015

Subject Code: 4420601 Date: 28-05-2015 **Subject Name: Discrete Mathematics for Computer Science** Time: 10:30 am - 01:00 pm **Total Marks: 70 Instructions:** 1. Attempt all questions. 2. Make suitable assumptions wherever necessary. 3. Figures to the right indicate full marks. Define poset. When is a poset said to be a lattice? Draw Hasse diagrams of 07 **Q.1** (a) following posets and examine which of them are lattices.  $(a) < P(S), \subseteq >, S = \{a, b, c\}$ (b) <{1, 2, 3, 12, 18}, D >  $(c) < \{1, 2, 3, 6\}, D >$  $(d) < S_{16}, D >.$ (b) Define: Boolean Algebra. Find all Sub Boolean Algebra of Boolean Algebra 07  $<S_{30}$ , , V, =0,1>. 04 **Q.2** (a) (1) Draw Hasse- diagram for the following : i)  $< S_{105}, D >$ ii)  $< S_{70}D >$ (2) With proper justification give an example of 03 a) A bounded lattice which is complemented but not distributive. b) A bounded lattice which is distributive but not complemented. c) A bounded lattice which is both distributive and complemented. Define: **(b)** 07 i) Join irreducible elements. ii) Atoms of a Boolean algebra. Determine Join-irreducible elements and atoms of following Boolean algebra also draw the Hasse Diagram: i)  $(S_{210}, D)$ ii)  $\langle P(S), , \cup, ', , S \rangle$  where  $S = \{a, b, c\}$ OR Define Lower bound and Upper bound. Let  $P = \langle 3, 5, 9, 15, 24, 45 \rangle$ , 07 **(b)** D> be a poset. Draw the Hasse diagram. Find i) maximal element. & minimal element. ii) the greatest and least element. iii) the lower bounds of  $\{3, 5\}$ , if any & the upper bound of  $\{9, 15\}$ , if any iv) GLB of {15, 45} & LUB of {3, 9, 15}.

Q.3 (a) Show that the lattice  $\langle S_n, D \rangle$  for n=100 is Isomorphic to the direct product of 07 lattice for n=4 & n=25.

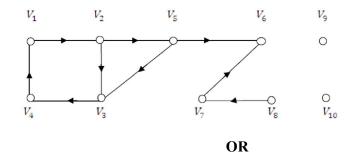
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(b) Use the Quine-McCluskey algorithm to find the prime implicants of the 07 expression:  $f(a, b, c, d) = \hat{U}(0, 1, 4, 5, 9, 11)$ . Also obtain a minimal expression for the same.

## OR

Q.3	(a)	Use K-map to find a minimal SOP expression for the function given by $\hat{U}$ (0,1,2,3,6,7,13,14) in four variables w,x,y,z.	07
	(b)	<ul> <li>(1) Explain Stoneøs representation theorem by giving a suitable example.</li> <li>(2) Obtain the sum-of-products canonical form of the following Boolean expressions: <ul> <li>i) (x1 ⊕ x2) ⊕ (x1 * x3)</li> <li>ii) (x1 * x2) ⊕ x3</li> </ul> </li> </ul>	03 04
Q.4	(a)	<ol> <li>(1) Define Sub-lattice. Write any four sublattices of (S<sub>12</sub>, D).</li> <li>(2) Describe the application of Boolean algebra to Relational Database.</li> </ol>	03 04
	(b)	(1) Show that in a group $\langle G, * \rangle$ if for any a , $b \in G$ , $(a * b)^2 = a^2 * b^2$ , then $\langle G, * \rangle$ must be abelian.	03
		(2) Show that $\langle \{1,4,13,16\}, x_{17} \rangle$ is a subgroup of $\langle Z_{17}^*, x_{17} \rangle$ . OR	04
Q.4	<b>(a)</b>	Show that the set of all positive rational number forms an abelian group under the composition defined by $a * b = ab/2$ .	07
	(b)	Define Cyclic Group. Prove that $\langle Z_7^*, x_7 \rangle$ is a group. Also find generators of this group.	07
Q.5	(a)	Define : Forest, Binary Tree, Node Base, Cycle, Elementary Path, Isolated Node, Graph.	07

(b) Define node base of a simple digraph. Find reachablility set of all nodes for 07 the following diagraph.



Q.5 (a) Define: Isomorphic Graph, Sling and Weighted Graph. State weather the following digraphs are isomorphic or not:

07



(b) Define: Directed tree and its leaf. Draw the graph of the tree represented by (A(B(C(D)(E)))(F(G)(H)(J))(K(L)(M)(N(P)(Q(R))))).
 Obtain the binary tree corresponding to it.

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