Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA – SEMESTER - I • EXAMINATION – WINTER 2012

Subject code: 610003 **Date: 05/01/2013**

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 02:30 pm – 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Prove that $<A,*_{17}>$ is group, where A = {1,4,13,16} and operation $*_{17}$ represents multiplication modulo 17. **07**

(b) Verify validity of given argument without using truth table: **07**
"If the meeting will be either in room 302 or 306 then we can find that the meeting is not in 302".

Q.2 **(a)** If A={1,2,3,4} and R={ <x,y> | x>=y}. Verify! reflexive, symmetric and transitive for given set of relation R. **07**

(b) Define Tautology. Prove that p -> (p V q) is tautology without constructing truth table. **07**

OR

(b)

(i) Prove that the product of an odd integer and an even integer is even. **03**

(ii) Write rule of Disjunctive Syllogism. Prove it using truth table. **04**

Q.3 **(a)** Define: Normal-Subgroup. Let $<Z_6, +_6>$ be the group and H = {0, 3} be the subgroup of Z_6. Verify that H is normal subgroup of Z_6. **07**

(b) Show that (Using laws) **04**

(i) $[\, a * (b' \oplus c)\,]' * [\, b' \oplus (a * c')'\,]' = a * b * c'$

(ii) $(x \oplus y) * (x' \oplus y) = y$ **03**

OR

Q.3 **(a)** Let the permutations of elements of {1, 2, 3, 4, 5, 6} be given by **07**

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 4 & 5 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 5 & 4 & 6 \end{pmatrix}$$

Find AB and solve the equation AX = B.

(b) Find a minimal sum-of-product form using K-map **07**

(i) $\alpha(x, y, z) = xyz + xyz' + x'yz' + x'y'z$

(ii) $\alpha(x, y, z) = xyz + xyz' + xy'z + x'yz + x'y'z$

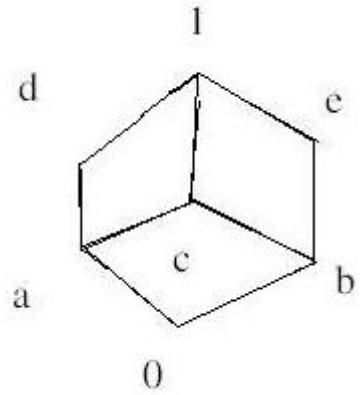
Q.4 **(a)** Define: Sub-Lattice. Show that S_6 is sub lattice of lattice $<S_{30}, D>$. **07**

(b) Draw Hasse diagram and find cover of each element of $< L^2 , \leq >$, where $\leq$ means (a, b) R (c, d) iff $a \leq c$ and $b \leq d$. L^2 means L x L. L={0,1} **07**

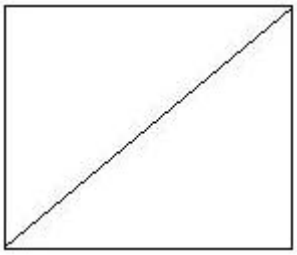
OR

Q.4 **(a)** Define: Lattice. Prove that, S_8 is lattice and draw Hasse diagram of S_8. **07**

(b) Define: Join-irreducible, Meet-irreducible, Atoms and Anti-Atom. Determine the join-irreducible, meet-irreducible, atoms and anti-atoms for given lattice. **07**



Q.5 **(a)** Define: Sub Graph. Find all possible sub graphs for the given graph **07**

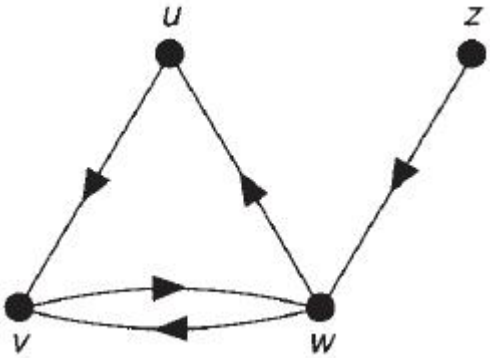


(b) Draw di-graph and find in-degree and out-degree of each vertex from the given adjacency matrix. Using adjacency matrix, find total numbers of path of length 1 and 2 between each vertex. **07**

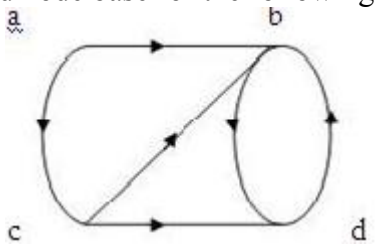
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

OR

Q.5 **(a)** State handshaking theorem. Verify handshaking theorem for the given graph **07**



(b) Define: Node Base. Find node base for the following graph: **07**



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