

GUJARAT TECHNOLOGICAL UNIVERSITY**MCA - SEMESTER-I • EXAMINATION – WINTER 2013****Subject Code: 2610003****Date: 23-12-2013****Subject Name: Discrete Mathematics for Computer Science (DMCS)****Time: 02:30 pm TO 05:00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Let $X = \{1, 2, 3, 4, 5\}$ and R, S, T be relations on X as follows :

$$R = \{(x, y) / x + y = 5\} \quad S = \{(1, 2), (3, 4), (2, 2)\} \quad T = \{(4, 2), (2, 5), (3, 1), (1, 3)\}$$

- (i) Write properties of R . 03
- (ii) Write matrix of R . 01
- (iii) Find $S \circ T, R \circ S$ and $S \circ R$. 03

- (b) (i) Define a group. Show that $(Z_6, +_6)$ is a group. Is it cyclic? 03
- (ii) Let $(G, *)$ be a group. Let $|G| = 5$. How many subgroups are there of G ? Why? 01
- (iii) Write (a) degree and (b) order of permutation group of (S_3, \diamond) . Is this group abelian? Justify your answer. 03

Q.2 (a) (i) Define a partial order relation. Let X be the set of positive integers and R be the relation " xDy " where D stands for "divides". Show that D is a partial order relation on X . 04

- (ii) Define an equivalence relation. Given a set $S = \{1, 2, 3, 4, 5, 6\}$. Find the equivalence relation on S which generates the partition $\{\overline{1, 2}, \overline{3, 4, 5, 6}\}$. Draw the graph of the relation. 03

- (b) (i) Draw Hasse Diagram of the following posets. 04
- (a) (S_{45}, D) (b) (S_{32}, D) .

- (ii) Show that the relation "congruence modulo m " given by $\equiv = \{(x, y) / (x - y) \text{ is divisible by } m\}$ over the set of positive integers is an equivalence relation. 03

OR

- (b) (i) Give an example of a set X such that $(\rho(X), \subseteq)$ is a totally ordered set. 02
- (ii) Give a relation which is both, a partial order relation and an equivalence relation on a set. 02
- (iii) Write a short note on applications of relations to database theory. 03

Q.3 (a) (i) Let P and Q be two statements. Write (a) converse and (b) contra positive of implication $P \rightarrow Q$. 02

- (ii) What is the truth value of the quantification $\exists xP(x)$? The domain of the discourse is the set of all real numbers. 02

$$(a) P(x): x+1=1 \quad (b) P(x): (x+1)^2 < 0.$$

- (iii) Test the validity of the logical consequence : 03

- (1) All drivers take a driving test. (2) Tom did not take the driving test.
- (3) Therefore Tom is not a driver.

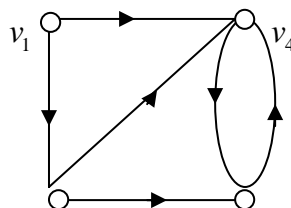
- (b) (i) Define an abelian group. Show that in a group $(G, *)$, if for any $a, b \in G$, $(a * b)^2 = a^2 * b^2$ then $(G, *)$ must be abelian. 03
- (ii) Define left coset of a subgroup H in a group G , determined by the element $a \in G$. Find the left cosets of $\{[0], [3]\}$ in the group $(Z_6, +_6)$. 02
- (iii) Define kernel of a group homomorphism g from a group $(G, *)$ to (H, Δ) . Show that it is never empty. 02

OR

- Q.3** (a) (i) Determine the truth value of each of the following statement. 02
- (a) Either today is Monday or $\sqrt{7}$ is a real number.
- (b) If Mickey is in Florida, then 17 is an odd integer.
- (ii) What is the truth value of the quantification $\forall x P(x)$? The domain of the discourse is the set of all positive integers. 02
- (a) $P(x) : (x+1)(x+2)$ is an even integer.
- (b) $P(x) : (x+1) > x$
- (iii) Test the validity of the logical consequences. 03
- Everyone who studies logic, is good in reasoning.
Lata is good in reasoning.
Therefore, Lata studies logic.
- (b) (i) Define an abelian group. Show that if every element in a group is its own inverse, then the group must be abelian. 03
- (ii) Show that $(\{[1], [4], [13], [16]\}, X_{17})$ is a subgroup of (Z_{17}^*, X_{17}) . 02
- (iii) Show that every subgroup of a cyclic group is normal. 02
- Q.4** (a) (i) Define a sublattice. Show that every interval of a lattice is a sublattice. 02
- (ii) Show that in a lattice if $a \leq b$ and $c \leq d$, then $a * c \leq b * d$ 02
- (iii) Define (a) A complemented lattice. 03
- (b) A distributive lattice.
- Give one illustration of a lattice, which is complemented and distributive both.
- (b) (i) In any Boolean Algebra, show that $a = b \Leftrightarrow ab' + a'b = 0$ 03
- (ii) Use Karnough map representation to find a minimal sum-of-products expression $f(a, b, c, d) = \sum(5, 7, 10, 13, 15)$ 04

OR

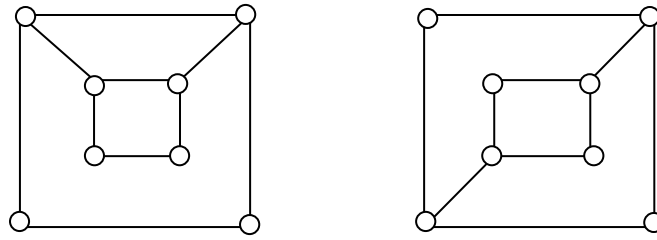
- Q.4** (a) (i) Define direct product of two lattices. Show that the lattice (S_{36}, D) is isomorphic to the direct product of lattices (S_4, D) and (S_9, D) . 05
- (ii) Define a complemented lattice. Which of the two lattices (S_n, D) for $n = 42$ and $n = 45$ are complemented? 02
- (b) (i) In any Boolean algebra, show that $a = 0 \Leftrightarrow ab' + a'b = b$. 03
- (ii) Use the Quine Mc Clusky method to simplify the sum-of-products expression $f(a, b, c, d) = \sum(10, 12, 13, 14, 15)$ 04
- Q.5** (a) (i) Give three different elementary paths from v_1 to v_3 for the digraph given in Figure -1. What is the shortest distance between v_1 and v_3 ? 03



v_2 v_3 Figure-1

- (ii) Define isomorphic graphs. State whether the following digraphs are isomorphic or not. Justify your answer.

04



- (b) (i) Define Reachable set of a node v . Find the reachable sets of $\{v_1\}$, $\{v_3\}$ and $\{v_5\}$ for the digraph given in Figure-2.

03

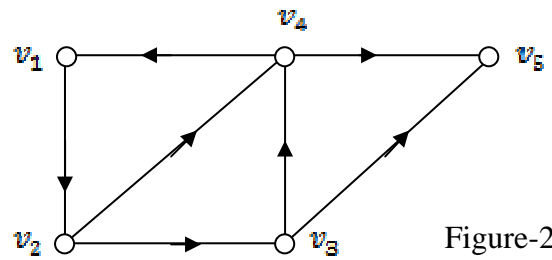


Figure-2

- (ii) Define: (a) A Binary tree (b) A complete m-ary tree

02

- (iii) State whether digraphs given in Figure-1 and Figure-2 are strongly, weakly or unilaterally connected.

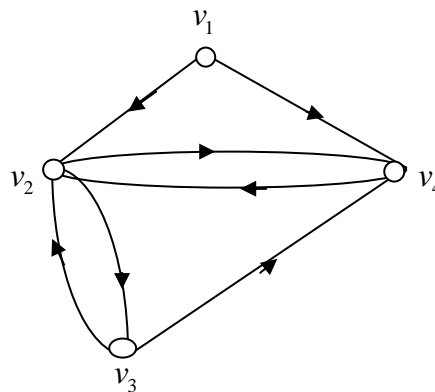
02

OR

- Q.5** (a) (i) Define node base of a simple digraph. Prove that in an acyclic simple digraph a node base consists of only these nodes whose indegrees is zero. (ii) Obtain the adjacency matrix A of the following digraph. Find the elementary paths of lengths 1 and 2 from v_1 to v_4 . Verify the results by calculating A^2 .

03

04



- (b) (i) Define a complete binary tree. Show that in a complete binary tree the total number of edges is given by $2(n_i - 1)$, where n_i is the number of terminal nodes. (ii) Define Isomorphic graphs. What are the necessary conditions for two graphs to be isomorphic? Are they sufficient also? Justify your answer.

03

04
