Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Remedial Examination April 2010

Subject code: 711403

Subject Name: Statistical Analysis & Numerical Methods

Date: 08 / 04 / 2010 **Time: 12.00 noon – 02.30 pm**

Total Marks: 60

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find the real root of the equation $f(x) = xe^x - 2 = 0$, which lies between 0.8 and 0.9 correct to three decimal points. **06**

(b) Solve the system of equations **06**

$$2x_1 + x_1 + x_1 + x_1 = -10$$
$$4x_1 + 2x_3 + x_4 = 8$$
$$3x_1 + 2x_2 + 2x_3 = 7$$
$$x_1 + 3x_2 + 2x_3 - x_4 = -5$$

using the Gauss elimination method with partial pivoting.

Q.2 **(a)** Derive Gauss-Legendre two point integration formula with error constant and error term. Hence, evaluate $\int_{-1}^{1} \frac{dx}{x+3}$. **06**

(b) For the following system of equations **06**

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

show that the Jacobi iteration scheme converges. Also, starting with $\bar{x}^{(0)} = \bar{0}$, iterate the system three times.

OR

(b) Obtain the cubic spline approximation for the function defined by the data **06**

x	0	1	2	3
f(x)	1	2	33	244

with $M(0) = 0$ and $M(3) = 0$. Hence find an estimate of $f(2.5)$.

Q.3 **(a)** Find the inverse of the matrix **06**

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

using LU-decomposition with Crout's method.

(b) The Hermite polynomials are given by **06**

$$H_{n+1} = (-1)^{n+1} e^{x^2} \frac{d^{n+1}}{dx^{n+1}} \left(e^{x^{-2}}\right)$$

Find the Hermite polynomials upto degree two. Also, find the nodes for the two-point Gauss-Hermite quadrature method.

OR

Q.3 **(a)** Using Householder's transformation reduce the matrix **06**

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

into a tri-diagonal matrix.

(b) Solve using Cholesky's method **06**

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Q.4 **(a)** Calculate the first four moments of the following distribution about the mean **06**

x	0	1	2	3	4	5	6	7
f	1	8	28	56	70	56	28	8

(b) Let X be a random variable with the following probability distribution **06**

X	-3	6	9
$p(X = x)$	1/6	½	1/3

Find the expectation $E(X)$, variance $V(X)$. Also, using the laws of expectations, evaluate $E(2X+1)^2$.

OR

Q.4 **(a)** A problem of statistics is given to three students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}$, respectively. What is the probability that the problem will be solved if all of them try independently? **06**

(b) In a bolt factory machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4 and 2 percents are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine A? **06**

Q.5 **(a)** Ten coins are thrown simultaneously. Find the probability of getting **06**

(a) exactly 8 heads.
(b) at least 9 heads.
(c) at the most 1 head.

(b) X is normally distributed with mean 12 and standard deviation 4. **06**

(a) find out the probability of

(i) $X \geq 20$ (ii) $X \leq 20$ (iii) $0 \leq X \leq 12$

(b) find x, when $p(X \geq x) = 0.24$

OR

Q.5 **(a)** Twenty people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survived rate, if attacked by this disease, is 85% in favour of the hypothesis that it is more, at 5% level, using the large sample test? **06**

(b) Fit a Poisson distribution to the following data and test the goodness of fit. **06**

x	0	1	2	3	4	5	6
f	275	72	30	7	5	2	1

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