seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Examination January 2010

Subject code: 710102

Subject Name: Quantum Computing & Algorithm Analysis

Date: 22 / 01 /2010 **Time: 12.00- 2.30 pm**

Total Marks: 60

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. ***R* denotes the set of real numbers, R^+ the set of non negative real numbers. R denotes the relation defined on set.**

Q-1

(a) **06**

1. Say whether the indicated function is one to one and say what its range is.
   m : $R^+ \rightarrow R^+$ defined by m(x) =min(x,2)
2. Is relation R an equivalence relation?
   A is the set of all infinite sequences $x=x_0x_1x_2\ldots$ of 0's and 1's. For two such sequences x and y, xRy if there exist an integer k so that $x_i=y_i$ for every $i\geq k$.
3. Find an LU decomposition of the matrix

$$\begin{bmatrix} 5 & 0 & 4 \\ 0 & 1 & 6 \\ 1 & 3 & 0 \end{bmatrix}$$

(b) **06**

1. Define : (1) Hermitian (2) Unitary operator (3) Normal operators
2. Explain three postulates of quantum mechanics
3. Explain important properties of trace.

Q-2

(a) **06**

1. Let S be a finite set, and let R be an equivalence relation on S × S. Show that if in addition R is antisymmetric, then the equivalence classes of S with respect to R are singletons.
2. Why amortized analysis is used? Perform the aggregate analysis on stack operations to find amortized cost.

(b) **06**

Let

$$|\Psi\rangle = \sqrt{\frac{3}{7}}\,|0\rangle + \frac{2}{\sqrt{7}}\,|1\rangle$$

(i) Write down the density matrix in the $\{|0\rangle, |1\rangle\}$ basis.

(ii) Determine whether or not this is a pure state.

(iii) Write down density matrix in the $\{|+\rangle, |-\rangle\}$ basis, show that Tr (ρ) =1 still holds and determine if you still obtain the same result as in part (ii).

OR

(b) **06**

Let S={a, b, c, d, e, f, g} be a collection of objects with benefit weight values as follows: a:(12,4), b:(10,6) , c:(8,5), d:(11,7), e:(14,3), f:(7,1), g:(9,6). What is an optimal solution to the 0-1knapsack problem for S assuming that knapsack can hold objects with total weight 18. How does the solution differ from fractional knapsack problem?

Q-3

(a) **06**

A three-state system is in the state

$$|\Psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|2\rangle$$

Write down the necessary projection operators and calculate the probabilities Pr(0), Pr(1), and Pr(2).

(b) **06**

Draw the frequency table and Huffman tree for the following string:

"dogs do not spot hot pots or cats"

Generate the Huffman code and write down the Huffman coding algorithm.

OR

Q-3

(a) Find the eigenvalues of **06**

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

(b) **06**

Explain the Divide and Conquer strategy. Use the Divide and Conquer algorithm, to compute 10110011×10111010 in binary. Show your work.

Q-4

(a) Draw a simple, connected , weighted graph with 8 vertices and 16 edges, each with unique edge weights. Identify one vertex as a "start" vertex and illustrate a running of Dijkstra's algorithm on this graph and write the algorithm. **06**

(b) **06**

The qubit trine is defined by the states

$$|\Psi_0\rangle = |0\rangle$$

$$|\Psi_1\rangle = -\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle$$

$$|\Psi_2\rangle = -\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

Find states $|\bar{\Psi}_0\rangle, |\bar{\Psi}_1\rangle, |\bar{\Psi}_2\rangle$ that are normalized, superposition states of $\{|0\rangle, |1\rangle\}$, and orthogonal to $|\Psi_0\rangle, |\Psi_1\rangle, |\Psi_2\rangle$ respectively.

OR

Q-4

(a) Write Euler tour traversal of a binary tree algorithm and Explain with example. **06**

(b) Suppose **06**

$$|\psi\rangle = 3|0\rangle - 2i|1\rangle$$

$$|\phi\rangle = |0\rangle + 5|1\rangle$$

(i) Show that these states obey the Cauchy-Schwarz and triangle inequalities.

(ii) Normalize the states.

Q-5

(a) A qubit is in the state **06**

$$|\Psi\rangle = \frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle$$

A measurement with respect to Y is made, Given that the eigen values of the Y matrix are ±1, determine the probability that the measurement result is ±1and the probability that the measurement result is -1.

(b) **06**

Explain how can you solve single source shortest paths with negative cost problem using dynamic programming?

OR

Q-5

(a) A system is in the state **06**

$$|\psi\rangle = \frac{1}{\sqrt{8}}|0\quad 0\rangle + \sqrt{\frac{3}{8}}|0\quad 1\rangle + \frac{1}{2}|1\quad 0\rangle + \frac{1}{2}|1\quad 1\rangle$$

(i) What is the probability that measurement finds the system in the state $|\phi\rangle = |0\quad 1\rangle$?

(ii) What is the probability that measurement finds the first qubit in the state $|0\rangle$? What is the state of the system after measurement?

(b) Prove that **06**

(i) Kruskal's method results in a spanning tree whenever a spanning tree exists.

(ii) The spanning tree generated with Kruskal's method is of minimum cost.

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