

Seat No.: \_\_\_\_\_

Enrolment No. \_\_\_\_\_

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**M.E Sem-I Examination January 2010**

**Subject code: 710401      Subject Name: Statistical Signal Analysis**

**Date: 20 / 01 / 2010**

**Time: 12.00-2.30pm**

**Total Marks: 60**

**Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a)** Give answer of following questions. **06**

1. Describe the use of probability theory in Communication Engineering.
2. Explain the meaning of statistical independent ness and uncorrelated ness of two random variables. Out of these two parameters of random variable which is the stronger notion of independent ness.
3. A uniformly distributed Random variable X has Probability Density Function (PDF)  $f_X(x) = 1/2\pi, 0 \leq X \leq 2\pi$ . Find the expected value of  $\cos(x)$  (i.e.  $E[\cos(x)]$ ) and expected value of  $x^2$  (i.e.  $E[x^2]$ ).

**(b)** Give answer of following questions. **06**

1. Someone has given you the Cumulative Distribution Function(CDF) of random variable as,

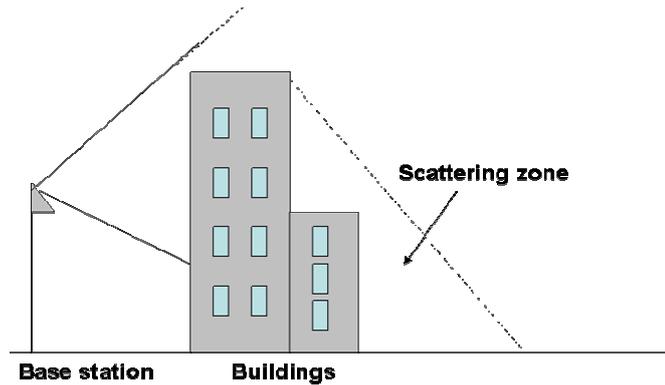
$$F_X(x) = \begin{cases} 1 - ae^{-x/b} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

As you are expert in Statistical Signal Analysis (SSA) she asks you to verify whether  $F_X(x)$  is a valid CDF or not. Give your answer to her query.

2. The Probability Density Function (PDF) of exponential distributed random X is given as  $f_X(x) = \lambda e^{-\lambda x}$   $x \geq 0$  and  $\lambda > 0$ . Find the characteristics function of random variable X.

**Q.2 (a)** In mobile radio communication when any obstruction (e.g. like hill or tall building) **06**

comes in to the path of signal then the region behind obstruction is called shadow region and only diffracted signal will reach to mobile unit, as shown in figure below, If transmitted signal has Gaussian distribution with mean  $m_x$  and variance  $\sigma^2$  than find distribution of received signal in scattering zone. See the figure below.



[Hint: In scattering zone received signal random variable is  $Y$ , then  $Y=e^X$ . where  $X$  is the transmitted signal random variable]

(b) Give answer of following questions. 06

1. Let  $Z$  be the random variable with PDF  $f_z(z)=1/2$ . Let new random variable  $X$  and  $Y$  is defined as,  $X=Z$  and  $Y=Z^2$ .  $X$  and  $Y$  both are uncorrelated?  $X$  and  $Y$  both are statistically independent?

2. The PDF of continuous random variable  $X$  is given as

$$f_x(x) = \begin{cases} 3.1437e^{-x} & 1 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that  $X$  takes a value greater than 2 given that it has a value greater than 1.5.

**OR**

(b) Give answer of following questions. 06

1. Give the PDF for Gaussian distributed random variable and show that Gaussian PDF integrates to one.

2. A communication system accepts a positive voltage  $V$  as a input and outputs a voltage  $Y=\alpha V+N$ , where  $\alpha=10^{-2}$  and  $N$  is a Gaussian random variable with parameter mean ( $m$ ) = 0 and  $\sigma=2$ . Find the value of  $V$  that gives  $P[Y<0]=10^{-6}$ .

**Q.3 (a)** Give answer of following questions. 06

1. Show that  $E[x]$  for the random variable with CDF  $F_X(x)=1-1/x$ , for  $x<1$ , does not exist.

2. Define the joint density function of two random variable and state how to get the marginal density function from joint density.

(b) Give answer of following questions. 06

1. State the Chebyshev's inequality for random variable.

2. State and prove the central limit theorem.

**OR**

**Q.3 (a)** Give answer of following questions. 06

1. Show how Chebyshev's inequality is useful to decide the narrowness or broadness of PDFs.

2. Define characteristics function and Moment generating function of random variable.

- (b) Give answer of following questions. 06
- For vectored random variable  $\underline{X}$  of dimension of  $2 \times 1$  find the PDF if random variables are jointly Gaussian.
  - If  $Y=1/X^2$ , where  $X$  and  $Y$  both are random variables. Find the  $F_Y(y)$  in terms of  $F_X(x)$

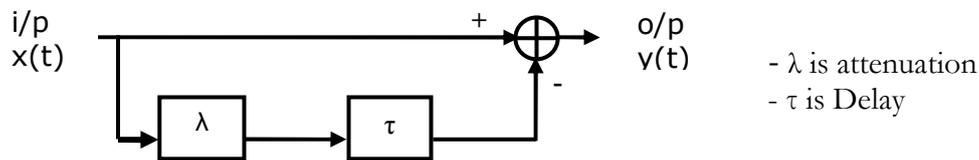
- Q.4 (a) Give answer of following questions. 06
- Let  $Y= 2X+3$ , if random variable  $X$  is uniformly distributed over  $[-1, 2]$ , find PDF of random variable  $Y$  and Plot PDF of random variable  $X$  and  $Y$ .
  - For random variable  $X$  and  $Y$  Joint density is given as,

$$f_{XY}(x, y) = \begin{cases} \frac{2(ax + by)}{a + b} & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute the conditional expectation  $E[X|Y=1/2]$

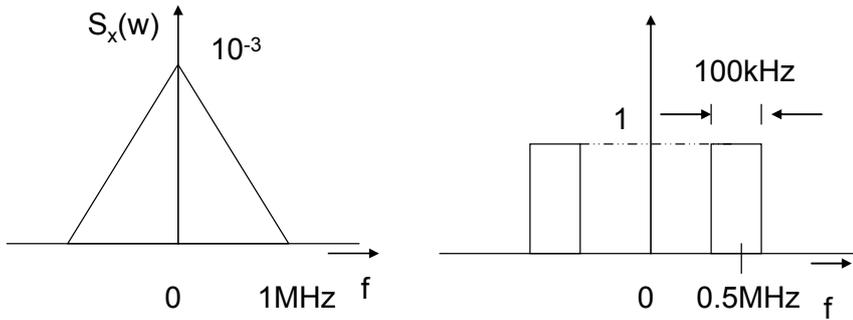
- (b) Give answer of following questions 06
- Consider a random process  $X(t) = U \cos(t) + V \sin(t)$  where  $U$  and  $V$  are independent random variables, each of which assumes the values  $-2$  and  $1$  with the probabilities  $1/2$  and  $2/3$ , respectively. Show that  $X(t)$  is Wide Sense Stationary random process.

- The autocorrelation function of a stochastic process  $X(t)$  is  $R_X(\tau) = 0.5N_0\delta(\tau)$ . Such process is called white process. If  $X(t)$  is the input to system as shown below find the power spectral density at output of the system.



OR

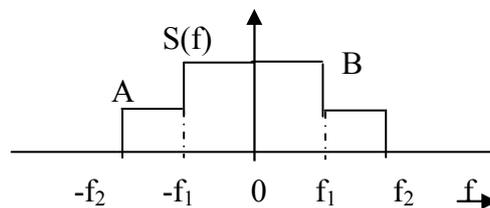
- Q.4(a) Give answer of following questions. 06
- If  $W(t)$  is a Wiener process, the will
    - $\alpha_1 W(t) + \alpha_2 W(t+T)$  be a Wiener process? Here  $T > 0$  and  $\alpha_1, \alpha_2$  are real constants.
    - $(W(t))^2$  be a Wiener process?
  - Give the classification of different types of random processes.
- (b) A random process  $x(t)$  with the PSD shown in figure below is passed, through a band pass filter with frequency response as shown in figure below. Determine the Mean square values of the quadrature components of the output process, Assume the center frequency in the representation to be  $0.5\text{MHz}$ . 06



**Q.5 (a)** Give answer of following questions

06

1. Find the autocorrelation function of random variable X whose power spectral density shown in figure below.



2. Let  $X_n$  be a sequence of uncorrelated random variable with zero mean and variance  $\sigma^2$ . Let

$$X(t) = \sum_{k=1}^n X_k e^{j\omega_k t}$$

$X(t)$  is a wide sense stationary process?

**(b)** Give answer of following questions.

06

1. Sketch the ensemble of the random process  $x(t) = at + b$  where b is a constant and a is an RV uniformly distributed in the range (-2,2). Just by observing the ensemble, determine whether this is a stationary or a non stationary process.
2. Explain the significance of convergence in mean square sense.

**OR**

**Q.5 (a)** Give answer of following questions.

06

1. Are the following covariance functions of a real stationary process (give reasons).

(a)  $\sin 4t$

(b)  $\delta(t-2)$

(c)  $R(t) = \begin{cases} 1 & |t| < 1 \\ 0 & |t| > 1 \end{cases}$

2. Given a random process  $x(t) = k$ , where k is an RV uniformly distributed in the range (-1,1).

(a) Sketch the ensemble of the random process

(b) Determine  $x(t)$

(c) Determine  $R_x(t_1, t_2)$

**(b)** Give answer of following questions.

06

1. Give the condition for which random process will become the Ergodic random process.
2. Define the point wise convergence and uniform convergence of random variable.

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