Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Examination January 2010

Subject code: 710904 **Subject Name: Optimization Techniques**

Date: 29 / 01 / 2010 **Time: 12.00- 2.30 pm**
Total Marks: 60

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) With the help of suitable examples explain the significance of (i) slack variables (ii) surplus variables and (iii) artificial variables, in the context of LP modeling. **06**

(b) A television manufacturing company has to decide on the number of 27 and 20 inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27inch and 10 of the 20 inch sets can be sold per month. The maximum number of work hours available is 500 per month. A 27 inch set requires 20 work hours and a 20 inch set requires 10 work hours. Each 27 inch sold produces a profit of Rs 6000 and each 20 inch produces a profit of Rs 4000. A wholesaler has agreed to purchase all the television sets produced if the numbers do not exceed the maxima indicated by the market research. **06**

Formulate a LP model for this problem and solve this model graphically

Q.2 (a) Write down the dual of the LP problem **06**

Maximize $z = 4x1 + 3x2 + 6x3$

Subject to $3x1 + x2 + 3x3 \leq 30$
$2x1 + 2x2 + 3x3 \leq 40$
$x1 \geq 0;\ x2 \geq 0;\ x3 \geq 0$

For the example given below check, graphically, whether it is a situation of multiple optimal solutions.

Maximize $z = 3x + 2y$

Subject to $3x + 2y \leq 18$
$x \leq 4$
$2y \leq 12$
$x, y \geq 0$

(b) Use Simplex method to solve the following problem **06**

Maximize $z = 4x1 + 3x2 + 6x3$

Subject to $3x1 + x2 + 3x3 \leq 30$
$2x1 + 2x2 + 3x3 \leq 40$
$x1 \geq 0;\ x2 \geq 0;\ x3 \geq 0$

OR

(b) Using Simplex method check whether a feasible solution is possible for the following problem **06**

Maximize $z = 5x1 + 3x2$

Subject to $2x1 + x2 \leq 1$

$x1 + 4x2 \geq 6$

$x1 \geq 0; x2 \geq 0$

Q.3 **(a)** Find the maxima of the function $f(x) = 2x_1+x_2+10$ subject to $g(x) = x_1+ 2x_2^2 =3$ by (i) method of constrained variation (ii) Lagrange multiplier method **06**

(b) The Simplex table of an LP model is given below. Complete the table and interpret the values given in the table. Is it an optimal solution? Specify the condition at which variable x3 will come in to the solution. Also write down the solution of the **dual** of the LP model. **06**

C_j			16	10	0	0	0	0	0
C_j	Product mix	Solution	x1	x2	x3	s1	s2	s3	s4
0	s1	785	0	0	3	1	0	-1/2	-1/2
0	s2	2355	0	0	0	0	1	-1/2	-1/2
17	x2	1185	0	1	1	0	0	1/2	-1/2
16	x1	30	1	0	0	0	0	0	1
	Zj								
	Zj-cj								

OR

Q.3 **(a)** Explain the steps involved in the following methods for solving LP problems (i) Two-phase method and (ii) Big M method **06**

(b) Maximize $f(x) = (x-1)^2$ **06**

Subject to $-2 \leq x \leq 4$

By applying Kuhn-Tucker conditions

Q.4 **(a)** Solve the following LP problem using the Branch and Bound method **06**

Maximize $z = 3x1 +4x2$

Subject to $7x1 + 11x2 \leq 80$

$3x1 - x2 \leq 12$

$x1, x2 \geq 0$ & integer

(b) Find the second order Taylors series approximation of the function $f(x_1, x_2, x_3) = x_2^2x_3 + x_1e^{x3}$ at the point (1,0,-2) **06**

OR

Q.4 **(a)** Solve the following LP problem using the cutting plane method **06**

Maximize $z = 3x1 +4x2$

Subject to $7x1 + 11x2 \leq 80$

$3x1 - x2 \leq 12$

$x1, x2 \geq 0$ & integer

(b) The profit/ acre of a farm is given by $20x_1 + 26x_2 + 4x_1x_2 - 4x_1^2 - 3x_2^2$, where x_1 and x_2 denote, respectively the labor cost and the fertilizer cost. Find the value of x_1 & x_2 to maximize profit. **06**

Q.5 **(a)** With the help of suitable example explain the following terms in the context of game theory (i) saddle point (ii) pure and mixed strategy (iii) Dominance in games **06**

(b) Reliability of an electronic device made of 4 components (failure of any component cause failure of the device i.e. they are connected in series) is to be improved by installing standby units to each component. Cost of the device should not exceed Rs 20,000. The reliability (R) and cost (C, in thousands of Rs) data are given below **06**

No of Units	Component 1		Component 2		Component 3		Component 4	
	R	C	R	C	R	C	R	C
1	0.6	3	0.7	4	0.5	3	0.6	3
2	0.7	4	0.8	6	0.6	5	0.8	5
3	0.8	5	0.9	8	0.9	6	0.9	7

Use dynamic programming to solve the problem

OR

Q.5 **(a)** For the following payoff table, use sub-game method to determine the value of the game and the optimal mixed strategy for each player according to the minimax criterion **06**

		Player 2		
	Strategy	1	2	3
Player 1	1	4	3	1
	2	0	1	2

Explain the basic philosophy of dominance method used to solve game theory problems

(b) Explain the characteristics of dynamic programming **06**

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