

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY**M.E Sem-I Examination January 2010****Subject code: 711403****Subject Name: STATISTICAL AND NUMERICAL ANALYSIS****Date: 25 / 01 / 2010****Time: 12.00 – 2.30 pm****Total Marks: 60****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Write the Vandermonde determinant for the problem of interpolating a function $f(x)$ defined over $[a, b]$ with nodes at $a = x_0, x_1, x_2, x_3 = b$ to obtain polynomial $P(x)$ satisfying the conditions $P(x_i) = f(x_i)$; $i = 0, 1, 2, 3$. Hence establish the existence of such a polynomial for $x_i \neq x_j$, $i, j = 0, 1, 2, 3$. **[02]**

(b) Approximate $\text{erf}(0.08)$ by linear interpolation in the given table of correctly rounded values. Estimate the total error. **[05]**

x	0.05	0.10	0.15	0.20
f(x)	0.05637	0.11246	0.16800	0.22270

Here, $\text{erf}(0.08) = \frac{2}{\sqrt{\pi}} \int_0^{0.08} e^{-t^2} dt$ is the error function and the error of interpolation,

using the data $(x_0, f(x_0))$, $(x_1, f(x_1))$ for the function is maximum $\frac{(x_1 - x_0)^2}{2\sqrt{2\pi}e}$.

(c) Define cubic spline. Obtain the cubic spline approximation valid in $[3, 4]$ for the function given in the tabular form **[05]**

x	1	2	3	4
f(x)	3	10	29	65

under the natural spline conditions

$$f''(1) = M(1) = 0 \text{ and } f''(4) = M(4) = 0$$

Q.2 (a) State a difference between Newton-Cotes methods of integration & Gauss quadrature methods. **[01]**

(b) Gauss-Chebyshev integration method is a method corresponding to weight **[05]**

$$w(x) = \frac{1}{\sqrt{1-x^2}} \text{ to evaluate integration of the form } \int_{-1}^1 \frac{f(x)}{\sqrt{1-x^2}} dx.$$

Derive Gauss-Chebyshev two-point method. Hence approximate the integral

$$\int_{-1}^1 \sqrt{1-x^2} \cos x dx$$

(c) Attempt any two:

[06]

(1) Laguerre polynomials are given by

$$L_{n+1} = (-1)^{n+1} e^x \frac{d^{n+1}}{dx^{n+1}} (e^{-x} x^{n+1}).$$

Find Laguerre polynomials upto degree two. Also, find the nodes for two-point Gauss-Laguerre quadrature method.

(2) State a strength and a weakness of Romberg Integration. Construct a Romberg table for $\int_0^2 \sqrt{4-x^2} dx$ with three rows.

(3) Write the general form of Adaptive quadrature formula, using Simpson's rule for $\int_a^b f(x) dx$. Hence, approximate $\int_0^2 \frac{1}{x^2 + \frac{1}{10}} dx$. Use the starting tolerance as

$$\varepsilon_0 = 0.001.$$

Q.3 (a) Solve the system of equations

[05]

$$\begin{bmatrix} 2 & 1 & -4 & 1 \\ -4 & 3 & 5 & -2 \\ 1 & -1 & 1 & -1 \\ 1 & 3 & -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \\ 2 \\ -1 \end{bmatrix}$$

using LU-decomposition method, assuming $u_{ii} = 1, i = 1(1)4$. Also find the condition number of A, where A is the coefficient matrix of the system and

$$\|A\| = \max_i \sum_{j=1}^4 |a_{ij}|.$$

(b) Solve the system of equations

[04]

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 5 & 2 \\ 5 & 4 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$$

using the Successive Over Relaxation (SOR) method. Determine W_{opt} and write down the iteration formula exactly.

(c) The average salary of male employees in a firm was Rs. 500 and that of females was Rs. 420. The mean salary of all the employees was Rs. 500, find the percentage of male and female employees. [03]

OR

Q.3 (a) Discuss Cholesky method of solving a square system of n-equations.

[05]

How many number of operations are required? Find the inverse of $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 4 & 6 \\ 2 & 6 & 29 \end{bmatrix}$,

using the method.

- (b) Reduce to the tri-diagonal matrix form, the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$, using the [04]

Householder's transformation.

- (c) A committee of 4 people is to be appointed from 3 officers of the construction management department, 4 officers from purchase department, 2 officers from the sales department and a chartered accountant. Find the probability of forming the committee in the following manner: [03]
- 1) There must be one from each category.
 - 2) It should have at least one from the construction management department.

- Q.4(a) The diameter of an electric cable, say X , is assumed to be a continuous random variable with probability distribution function $f(x) = 6x(1-x); 0 \leq x \leq 1$. [04]
- 1) Check that it is a probability distribution function.
 - 2) Determine a number 'b' such that $P(X < b) = P(X > b)$

- (b) Given the following table: [04]

x	0	1	2	3
P(x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{24}$	$\frac{1}{8}$

Compute $E(4X + 5)$ and $V(X)$.

- (c) The mean soil requirement to level one acre plot is 662 tractors with a standard deviation of 12 tractors. Assuming normal distribution, how many one acre plot in a batch of 1000 one acre plots would you expect to have soil requirement of [04]
- 1) over 700 tractors.
 - 2) below 650 tractors.

OR

- Q.4 (a) The life of a construction is a random variable having probability distribution function [04]

$$f(x) = \begin{cases} \frac{1}{20} e^{-\frac{x}{20}} & ; \text{for } x > 0 \\ 0 & ; \text{for } x \leq 0 \end{cases}$$

Find the probability that the construction will last

- 1) at least 10 years.
 - 2) any where between 15 to 20 years.
 - 3) at least 25 years.
- (b) In a mix prepared by two contractors A and B, the chance of a successful mix is in the ratio 3:2. Find A's chance of making a successful mix at least three times out of five mixes prepared. [04]
- (c) Find the most likely price in Ahmedabad to the price of Rs. five crore per 2000 sq ft office space from the following: [04]

	Mumbai	Ahmedabad
Average price(Rs. in crore per 2000 sq ft)	10	4
Standard deviation	0.5	0.7

Correlation coefficient between the prices of office space in the two cities is 0.8.

- Q.5(a)** A company has a head office at Surat and a branch at Rajkot. The personnel [04]
director wanted to know if the workers at the two places would like the
introduction of a new policy of work and a survey was conducted for this purpose.
Out of a sample of 500 workers at Surat 62% favoured the new policy. At Rajkot
out of a sample of 400 workers, 41% were against the new policy. Is any
significant difference between the two groups in their attitude towards the new
policy at 5% level of significance.

- (b)** The following table gives the number of accidents at seven different [04]
construction sites of a company. Find whether the accidents are uniformly
distributed over the sites.

Construction site	I	II	III	IV	V	VI	VII
Number of accidents	14	16	8	12	11	9	14

- (c)** In one sample of 8 observations, the sum of squares of deviations of the [04]
sample from the sample mean was 84.4 and in the other sample of 10
observations, it was 102.6. Test whether this difference is significant at 5% level,
given that the 5% point of F-distribution for $n_1 = 7$ and $n_2 = 9$, the degree of
freedom is 3.29.

OR

- Q.5(a)** An insurance company has claimed that the average life of insured building [05]
through it, is less than the average for all the remaining insurance companies
which is 30.5 years.
A random sample of 100 buildings insured by the company gave the following life
distribution:.

Life(to the year 2009)	16-20	21-25	26-30	31-35	36-40
Number of buildings	12	22	20	30	16

Calculate the arithmetic mean and standard deviation of this distribution and use
this value to test his claim at 5% level of significance, given that $Z(1.645) = 0.95$.

- (b)** The ages of 10 senior citizens of a township are founded to be [04]
70, 67, 62, 68, 61, 68, 70, 64, 64, 66 years.
Is it reasonable to believe that the average age is greater than 64 years? Test at 5%
significant level, assuming Student's t-distribution.

- (c)** Let x_1, x_2, \dots, x_n be a random sample from the uniform distribution function [03]

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} & ; 0 < x < \theta, \theta > 0 \\ 0 & ; elsewhere \end{cases}$$

Obtain the maximum likelihood estimation for θ .
