Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Examination January 2010

Subject code: 711503 **Subject Name: Advanced Solid Mechanics**

Date: 25 / 01 / 2010 **Time: 12.00 – 2.30 pm**

Total Marks: 60

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Drawing neat sketch for an element subjected to body forces, radial stresses, transverse stresses and shear stresses, derive the basic differential equations of equilibrium in 2-D Polar Coordinate System. **06**

(b) Derive Airy's stress function in Polar Co-ordinate System for an Axi-symmetric stress distribution. **06**

Q.2 **(a)** Explain soap film bubble analogy and derive the following equation with usual notations: **06**
$\nabla^2 (\phi/2G\theta) = \nabla^2 (sz/p) = -1$.
Comment on usefulness of membrane analogy for solution of complex problems in torsion.

(b) Drawing neat sketch for the element subjected to 3-D state of Cartesian stresses along with body forces, derive any one basic differential equation of equilibrium and write the other two equations. Also, write the solutions of stresses in terms of Airy's stress function for any 2-D state of stresses. **06**

OR

(b) Drawing neat sketch for the displacement of an element, derive the equation for various strain in Polar Coordinate System. **06**

Q.3 **(a)** For a solid circular disk of small uniform thickness, material density: ρ, rotating about the center with angular velocity: ω derive the expressions for radial stress and tangential stress. **06**

(b) For the following state of stresses, find the principal stresses using CARDAN's method and the direction cosines of any **ONE** principal stress. **06**
Normal stresses: $\sigma_{xx} = 450$ MPa, $\sigma_{yy} = -80$ MPa, $\sigma_{zz} = -50$ MPa, and
Shear stresses: $\tau_{xy} = 90$ MPa, $\tau_{yz} = 30$ MPa, $\tau_{zx} = -60$ MPa,

OR

Q.3 **(a)** In an infinitely long and wide plate of width: **2b** is subjected uni-axial stress along length, a circular hole of very small radius: **a** is made at the centre of plate, prove that along the inner circumference of the hole, stress changes the nature. Also, prove that the maximum stress will be three times the axial stress. **06**

(b) Is the following 2-D state of plane strain is possible? Check. **06**
$\varepsilon_X = 4x^3y + 3x^2 - 13.5x^2y^2 + 18y + 4$
$\varepsilon_Y = 4xy^3 + 6x - x^2 + 3y^2 + 5$
$\varepsilon_{XY} = \frac{1}{2}\gamma_{XY} = 2x^2 + 1.5y^2 + 4.5x^3y + xy + 4$

Q.4 **(a)** Derive the following equation with usual notations: **06**
$\varepsilon_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\,\mathrm{Cos}\,2\theta + \varepsilon_{xy}\,\mathrm{Sin}\,2\theta$

(b) For axi-symmetrical stress distribution in Polar Cordinate System, constant **B** of Airy's stress function: $\boldsymbol{\phi = A \ln r + B r^2 \ln r + Cr^2 + D}$ is required to be zero. Why? Also explain.the same. **06**

OR

Q.4 **(a)** Give characteristics of Airy's stress function. Is $\phi = A(y^4 - 3x^2y^2)$ representing Airy's stress function? Here, A is a constant. **06**

(b) An elastic body under the action of external forces has displacement field given by $\delta = (2x^2)\,\mathbf{i} - (5zy)\,\mathbf{j} + (2x^2z - 3y)\,\mathbf{k}$ in nm where x, y and z are expressed in mm. Determine all the strain components at (4, –2, 2). **06**

Q.5 **(a)** What is the concept of stability of structures? Give basis of stability of analysis for a slender straight column as well as column initially bent. **06**

(b) Derive equation of buckling load & deformation for the column with one end hinged & other fixed which produces structural instability. **06**

OR

Q.5 **(a)** Write the compatibility equation for 2-D strains. From which derive the equation $\nabla^4\phi = 0$, where ϕ represents Airy's stress function. **06**
For a 2-D state of displacements: $\mathbf{u = 5\,x^2\,y^{-5}}$ and $\mathbf{v = 0.02\,x^{-3}\,y^2}$, find state of stresses at a point (x, y) = (30, 60).

(b) Discuss effect of transverse shear on buckling of the beam & derive equation of critical load for the same. **06**

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