Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. 1ST Semester Remedial Examination –July- 2011

Subject code: 710107

Subject Name: Quantum Theory & Algorithm Design

Date:15/07/2011 **Time: 10:30 am – 01:00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define Pauli operators. **07**

The Pauli operators are given by

$$X=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Y=\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad Z=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

In the $\{|0\rangle, |1\rangle\}$ basis. Find the action of these operators on the basis stated by considering the column vector representation of the $\{|0\rangle, |1\rangle\}$ basis.

(b) Prove the theorem that an equivalence relation is the same as a partition. **07**

Q.2 **(a)** **07**

Suppose $\rho=\begin{bmatrix} \frac{1}{3} & \frac{i}{4} \\ -\frac{i}{4} & \frac{2}{3} \end{bmatrix}$

a. Is this a valid density matrix? If not, why not?
b. If this is a valid density matrix, does it represent a pure state or a mixed state?

(b) (i) Define surjection and injection with example. **04**

(ii) Show that the set of odd natural numbers is countable. **03**

OR

(b) (i) Write Prim's algorithm. **04**

(ii) Prove that the fractional knapsack problem has the greedy choice property. **03**

Q.3 **(a)** (i) Prove the following relations involving the trace operations: **04**

Tr(A+B)=Tr(A)+Tr(B)

Tr(λA)=λTr(A)

Tr(AB)=Tr(BA).

(ii) Write the postulate of measurements of Quantum Mechanics. **03**

(b) Prove that the Huffman algorithm has a greedy choice property. What is an optimal Huffman code for the following set of frequencies, based on first 8 Fibonacci numbers? **07**

a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21

OR

Q.3 **(a)** (i) A qubit is in the state $|\Psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}\,|1\rangle$ **04**

Using the projection operators P_0 and P_1. Indicate the probability of finding $|0\rangle$ and probability of finding $|1\rangle$ when a measurement is made.

(ii)Explain Heisenberg uncertainty principle. **03**

(b) How aggregate analysis of amortized analysis can be applied to stack operations and also for incrementing a binary counter? **07**

Q.4 **(a)** (i) How tensor product can be calculated when state vectors are represented as column vectors? **04**

(ii) Find the tensor product of Pauli matrices X and Z. **03**

(b) How insertion and deletion can be performed on binary search tree? **07**

Q.4 **(a)** (i) Write key properties of s density operator. **04**

(ii) Prove that for a mixed state $Tr(\rho^2)<1$. **03**

(b) For what purpose Dijkstra's algorithm is used and for what purpose Bellman Ford algorithm is used? Write Bellman Ford algorithm. **07**

Q.5 **(a)** Suppose $\rho=\begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix}$. Show that this is a valid density operator. **07**

(b) Explain Euler's tour with example. **07**

OR

Q.5 **(a)** Show that the eigenvectors of $Y=\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ are $|u_1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$, $|u_1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$. **07**

(b) Explain divide and conquer strategy and also show how divide and conquer strategy can be applied to merge sort? **07**

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