Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. IInd Semester–Remedial Examination – June/July- 2011

Subject code: 720110

Subject Name: Numerical Methods

Date:01/07/2011 **Time: 10:30 am – 01:00 pm**

Total Marks: 60

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Define absolute, relative and percentage errors. Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its absolute and relative errors. We have $\sqrt{3} = 1.732, \sqrt{5} = 2.236, and \sqrt{7} = 2.646.$ **06**

(b) Determine the root of $xe^x = 1$ correct to three decimal places using Bisection method. **06**

Q.2 (a) Determine the constants a and b by the method of least squares such that $y = ae^{bx}$ fits the following data: **06**

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

(b) Find a root of the equation $e^x - 3x^2 = 0$ using Newton-Raphson method correct to five decimal places. **06**

OR

(b) Find a real root of the equation $x^4 - 11x + 8 = 0$ using False-position method accurate to three decimal places. **06**

Q.3 (a) Solve the following equations by Gauss elimination method: **06**

$$2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16$$

(b) Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ and also solve the system of equations $AX = B$, where $B = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$. **06**

OR

Q.3 **(a)** Solve the following equations by Gauss-Seidal method: **06**

$x + 2y + z = 0, 3x + y - z = 0, x - y + 4z = 3.$

(b) Using Lagrange's interpolation formula find the value of y corresponding to $x = 10$ from the following data points: (5,380), (6,-2), (9,196), (11,508). **06**

Q.4 **(a)** Evaluate the integral $\int_0^{12} \frac{dx}{1+x}$, taking six intervals by using trapezoidal rule upto three significant figures. **06**

(b) Evaluate $\int_0^{1.2} e^x dx$ by using Simpson's one-third rule and Simpson's three-eight rule, taking $n = 6$. **06**

OR

Q.4 **(a)** Evaluate $\int_0^1 \sqrt{1 - x^2} dx$, correct to three decimal places by using trapezoidal and Simpson's one-three rules with $h = 0.25$. **06**

(b) The velocities of a car (running on a straight road) at intervals of 2 minutes are given below. Apply Simpson's three-eight rule to find the distance covered by the car. **06**

t(minutes)	0	2	4	6	8	10	12
v(km/hr)	0	22	30	27	18	7	0

Q.5 **(a)** Apply Euler's method to approximate the solution of the initial value problem $\frac{dy}{dx} = \frac{1}{2}y$ with $y(0) = 1$ in the interval $0 \leq x \leq 1$, using $h = 0.1$. **06**

(b) Use the second-order Runge-Kutta method with $h = 0.1$, to integrate $\frac{dy}{dx} = \sin y, y(0) = 1$ from $x = 0$ to 0.5. Keep four decimal places in the calculations **06**

OR

Q.5 **(a)** Use the modified Euler's method to find approximate value of $y(1.5)$ by solving the differential equation $\frac{dy}{dx} = 2xy$ with $y(1) = 1$. Take the step size $h = 0.1$. **06**

(b) Use the Runge-Kutta method of order four with $h = 0.1$ to find an approximate solution of $\frac{dy}{dx} = x^2 + y$ $y(0) = -1$ at $x = 0.1, 0.2$ and 0.4. **06**

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