Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-II Examination June 2011

Subject code: 720901 **Subject Name: Finite Element Method**

Date:22/06/2011 **Total Marks: 60** **Time: 10:30am-1:00pm**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Explain the basic steps to solve partial differential equation in approximate way by finite element method. **06**

(b) The governing equation for a fully developed steady laminar flow of a Newtonian viscous fluid on an inclined flat surface is given by **06**

$$\mu \frac{d^2 v}{dx^2} + \rho g \cos\theta = 0$$

Here μ is the viscosity, v is the fluid velocity, ρ is the density, g is acceleration due to gravity, θ is the angle between inclined surface and vertical. The boundary conditions are given by

$$\left.\frac{dv}{dx}\right|_{x=0} = 0 \quad \text{(zero shear stress condition)}$$

$$v(L) = 0 \quad \text{(no slip condition)}$$

Determine the velocity distribution $v(x)$ using weighted residual method.

Q.2 **(a)** Consider a 1 mm diameter, 50 mm long aluminum fin used to enhance heat transfer from a surface wall maintained at 300°C. The governing equation and boundary conditions are given by **06**

$$k\frac{d^2 T}{dx^2} = \frac{Ph}{A}(\Delta T)$$

$$\left.\frac{dT}{dx}\right|_{x=l} = 0$$

$$T(x=0) = T(wall) = 300^\circ C$$

Where k = 200 W/m/°C is the coefficient of thermal conductivity, P is the perimeter, A is the cross sectional area, h = 20 W/m^2°C is the convective heat transfer coefficient, ΔT is the temperature difference between the surface of the fin and ambient temperature which in present case is 30°C. Determine the Temperature distribution in the fin using Galerkin weighted residual method.

(b) Consider the bar shown in Fig. 1. An axial load of P = 200 x 10^3 N is applied as shown. Using the penalty approach for handling the boundary conditions, do the following. **06**

(a) Determine the nodal displacements.
(b) Determine the stresses in each material

(c) Determine the reaction forces

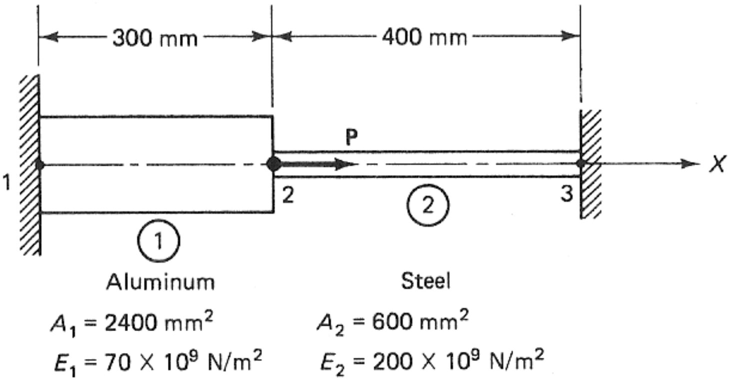

Fig.1

OR

(b) In Fig. 2, a load $P = 60 \times 10^3$ N is applied as shown. Determine the displacement field, stress and support reactions in the body. Consider $E = 20 \times 10^3$ N/mm^2. **06**

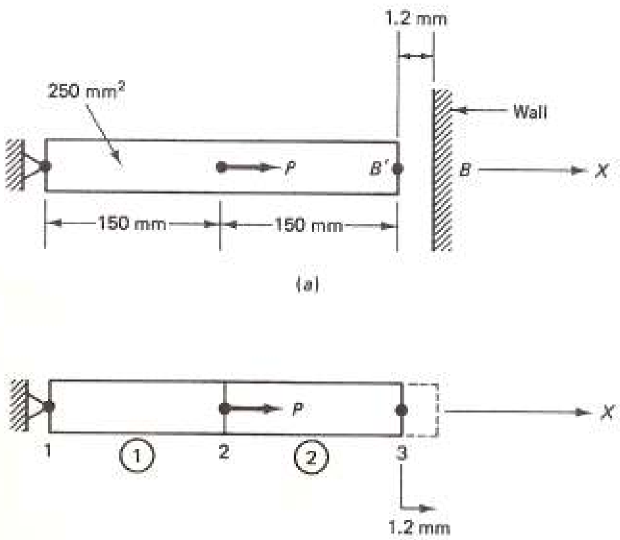

Fig. 2

Q.3 **(a)** An axial load $P = 300 \times 10^3$ N is applied at 20°C to the road as shown in Fig. 3. The temperature is then raised to 60°C. **06**

(a) Assemble stiffness and load matrices.

(b) Determine the nodal displacements and element stresses

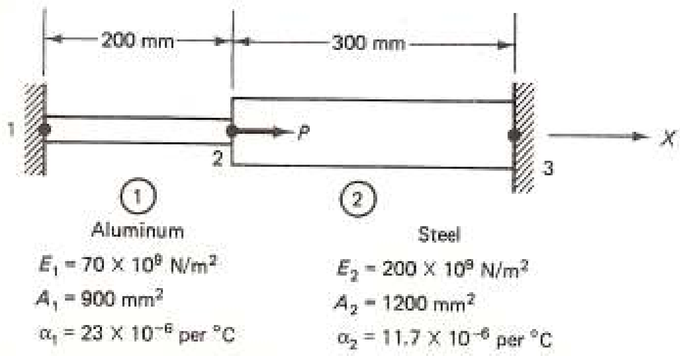

Fig. 3

(b) Consider the structure shown in the Fig. 4. A rigid bar of negligible mass, pinned at one end, is supported by steel rod and an aluminum rod. A load of $P = 30 \times 10^3$ N is applied as shown. **06**

(a) Model the structure using two finite elements. What are the boundary conditions for your model?

(b) Develop the modified stiffness matrix and modified load vector. Solve the equation for Q and then determine the element stresses.

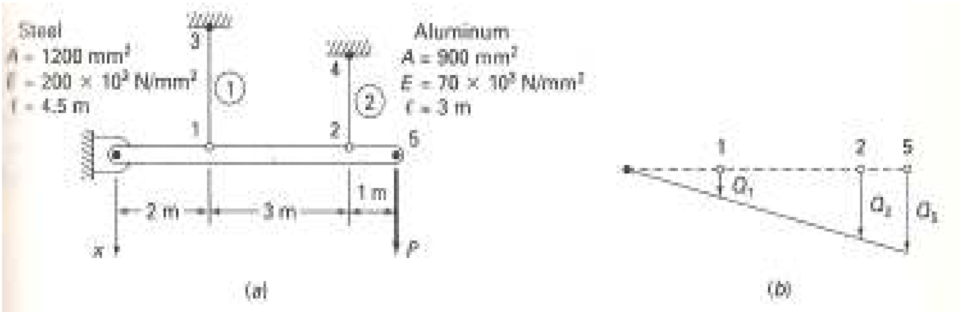

Fig.4

Table: 2 Elemental Details

Element No	Length in m	Area in mm^2	Modulus of Elasticity in N/mm^2
1 - Steel	4.5	1200	200,000
2 - Aluminum	3.0	900	70,000

OR

Q.3 **(a)** State the principle of minimum potential energy. Derive the equation of total potential for linear elastic solids. **06**

(b) Justify the following statements with reasons: **06**

(a) If the assumed trial solution of the given partial differential equation is the exact solution, then the weighted residual form is implicitly satisfied for any and every weighting function.

(b) The weighting function is subject to less stringent continuity requirement than the dependent field variable.

Q.4 **(a)** Justify the following statements with reasons: **06**

(a) A beam element can be considered simple line element, representing the neutral axis of the beam.

(b) The lumped force vector as against consistent nodal force vector does not guarantee work equivalence and thus one expects inferior results.

(b) Consider the frame structure made of steel as shown in Fig.5. Determine the deflections using the frame element. The elemental details are given in the Table 1 below. **06**

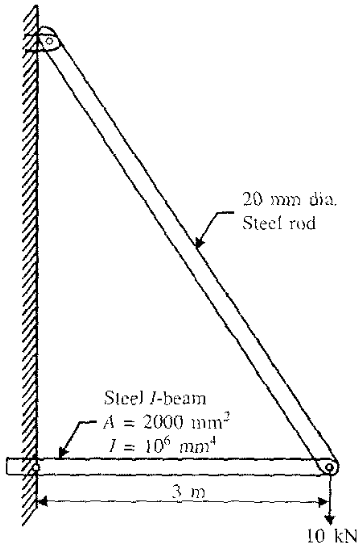

Fig 5

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Table .1 Elemental Details

Element No	Length in m	Area in mm^2	Modulus of Elasticity in N/mm^2	Moment of Inertia in mm^4
1	3.0	2000	200,000	10^6
2	5.0	314.16	200,000	7854

OR

Q.4 **(a)** Consider the three member truss shown in Fig. 6. All members of the truss have identical cross sectional area A and modulus of Elasticity E. The hinged support at point 1, 2 and 3 allow free rotation of the members about y axis (taken as positive into the plane of the paper). Determine the horizontal and vertical displacements at joint 3 and forces in each member of the structure. The governing homogeneous differential equation in usual notations is given as **06**

$$EA\frac{d^2u}{dx^2} = 0$$

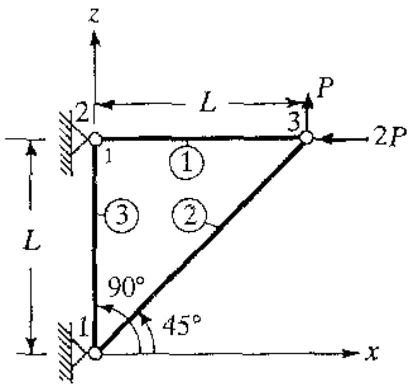

Fig.6

(b) Construct the weak form for the non linear Euler – Bernoulli partial differential equation of beam theory **06**

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) + kw = f \text{ for } 0 < x < l$$

$$w = \left(EI\frac{d^2w}{dx^2}\right) = 0 \quad \text{at } x = 0, l$$

Not that EI and f are functions of x and k is the foundation modulus and is to be considered constant.

Q.5 **(a)** Derive the shape functions for the following elements through Serendipity approach in natural coordinates **06**

(a) Four node quadrilateral element

(b) Three node triangular element

(b) Isoparametric elements are frequently used in finite element formulation whereas subparametric formulations are seldom used. – Justify the statement with proper reasoning. **06**

OR

Q.5 **(a)** Derive the shape functions for the three node constant strain triangle in Cartesian coordinates. **06**

(b) Construct the weak form for the non linear equation **06**

$$-\frac{d}{dx}\left(u\frac{du}{dx}\right) + f = 0 \text{ for } 0 < x < l$$

$$\left(u\frac{du}{dx}\right)\Big|_{x=0} = 0, u(1) = \sqrt{2}$$

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