Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Regular Examination January / February 2011

Subject code: 710107N

Subject Name: Quantum Theory & Algorithm Design

Date: 05 /02 /2011 **Time: 02.30 pm – 05.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** 1. Let f: N -> R defined as $f(x) = x^2 + 4x + 8$, then show that f: N -> A, where A is the range of f, is invertible. Also find f^{-1} 03

2. Design a divide and conquer MAXMIN algorithm that finds minimum and maximum out of n array element. 04

(b) Suppose **07**

$$|\Psi\rangle = 3|0\rangle - 2i|1\rangle \ , \ |\Phi\rangle = |0\rangle + 5\,|1\rangle$$

a) Show that these states obey the Cauchy-Schwarz and triangle inequalities.

b) Normalize the states.

Q.2 **(a)** (i) Does the matrix $\begin{bmatrix} 1/4 & (1-i)/4 \\ (1-i)/4 & 3/4 \end{bmatrix}$ represent a density operator? **03**

(ii) Two quantum states are given by **04**

a) $|a\rangle = \begin{bmatrix} -4i \\ 2 \end{bmatrix} \quad |b\rangle = \begin{bmatrix} 1 \\ -1+i \end{bmatrix}$

a) Find $|a+b\rangle$

b) Calculate $3\,|a\rangle - 2|b\rangle$

c) Normalize $|a\rangle, |b\rangle$

(b) (i) Find LU decomposition of following matrix. **03**

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 11 \\ 0 & 5 & -8 \end{bmatrix}$$

(ii) Which method of amortized analysis represents the prepaid work as credit associated with data structure as a whole? Explain the stack operations with this method **04**

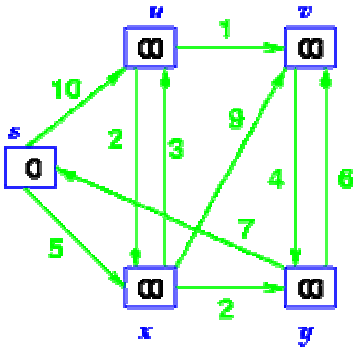
OR

(b) (i) Prove that Prim's algorithm finds a minimum spanning tree. **03**

(ii) Given: a set T of n tasks, each having:A start time, si and a finish time, fi (where si < fi) **04**

[1,4], [1,3], [2,5], [3,7], [4,7], [6,9], [7,8] (ordered by start) . Design a greedy solution to perform all tasks on minimum number of machines. Write pseudo code for the same.

Q.3 **(a)** (i) Given a graph, find the shortest paths using Dijkstra's algorithm. **03**



(ii) What is principal of optimality? How 0/1 knapsack problem be solved by dynamic programming? **04**

(b) A system is in the state **07**

$$|\Psi\rangle = \frac{1}{\sqrt{6}}|0\rangle + \sqrt{\frac{5}{6}}|1\rangle$$

A measurement is made with respect to the observable X. What is the expectation or average value?

OR

Q.3 **(a)** (i) Suppose that n=8, [w1,…,w8]=[100,200,50,90,150,50,20,80] and capacity c=400. When the greedy algorithm is used, in which order the containers are considered for loading? Prove that greedy algorithm generates optimal loadings. **03**

(ii) Write Kruskal's algorithm to find a minimum cost spanning tree in an n-vertex network. **04**

(b) Consider the following state vector: **07**

$$|\Psi\rangle = \sqrt{\frac{5}{6}}|0\rangle + \frac{1}{\sqrt{6}}|1\rangle$$

a) Is the state normalized?

b) What is the probability that the system is found to be in state $|0\rangle$ if Z is measured?

c) Write down the density operator.

d) Find the density matrix in the $|0\rangle, |1\rangle$ basis and show that Tr(ρ)=1

Q.4 **(a)** (i) Explain collision and overflow? How linear probing can help in handling the overflow? **03**

(ii) Given set of numbers and their frequency of use and create a huffman encoding for them: **04**

FREQUENCY	VALUE
5	1
7	2
10	3
15	4
20	5
45	6

(b) (i) Find $X \otimes Y|\Psi\rangle$, where **03**

$$|\Psi\rangle = (\,|0\rangle|1\rangle - |1\rangle|0\rangle\,)/\sqrt{2}$$

(ii) Explain eigenvalues, eigenvectors and characteristic equation. **04**

OR

Q.4 **(a)** (i) Show that for any finite set S, the power set 2^s has $2^{|s|}$ elements. **03**

(ii) Prove that an equivalence relation is the same as a partition. **04**

Q.4 **(b)** (i) If **03**

$$|\Psi\rangle = (\,|00\rangle + |11\rangle\,)/\sqrt{2}$$

Find $I \otimes Y|\Psi\rangle$,

(ii) How composite states are represented in quantum mechanics? **04**

Q.5 **(a)** (i) Define binary search tree. Explain insertion of an element in the binary search tree. **03**

(ii) How does breadth first search algorithm work? Let G be an arbitrary graph and v be any vertex of G. The breadth first search algorithm labels all vertices that are reachable from v(including vertex v) **04**

(b) Consider an ensemble in which 40% of the systems are known to be prepared in the state **07**

$$|\Psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}\,|1\rangle$$

And 60% of the systems are prepared in the state

$$|\theta\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

a) Find the density operators for each of these states, and show they are pure states. If measurements are made on systems in each of these states, what are the probabilities they are found to be in states $|0\rangle$ and state $|1\rangle$, respectively?

b) Determine the density operator for the ensemble.

OR

Q.5 **(a)** (i) Explain how the depth first search algorithm work? Let G be an arbitrary graph and v be any vertex of G. The depth first search algorithm labels all vertices that are reachable from v(including vertex v) **03**

(ii) Illustrate with example Euler tour traversal of a binary tree. **04**

(b) Find the change of basis matrix to go from the computational basis $\{|0\rangle, |1\rangle\}$ to the $\{|\pm\rangle\}$ basis **07**

$$|\Psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}\,|1\rangle$$

And the operator T= $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\Pi/4} \end{bmatrix}$ in the $\{|\pm\rangle\}$basis.

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