Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Regular Examination January / February 2011

Subject code: 710301N

Subject Name: Control Engineering

Date: 31 /01 /2011 **Time: 02.30 pm – 05.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Write a short note on Full Order Observers. **08**

(b) Consider the linear autonomous system **06**

$X(k+1) = \begin{bmatrix} 0.5 & 1 \\ -1 & -1 \end{bmatrix} x(k)$

Using direct method of Lyapunov, determine stability of the equilibrium state.

Q.2 **(a)** Show that the extremal for the functional, **08**

$\mathbf{J}(x) = \int_0^{\pi/2} (\dot{x}^2 - x^2)dt$

Which satisfies the boundry conditions $x(0) = 0$; $x(\pi/2) = 1$ is $x^*(t) = \sin t$.

(b) Prove that the state equation, $\dot{x}(t) = Ax(t) + Bu(t)$ is asymptotically stable if and only if all the Eigen values of matrix A have negative real parts. **06**

OR

(b) State and prove the sufficient condition of stability for the autonomous system, $\dot{x}(t) = f(x); f(0) = 0$ using Lyapunov's Stability theorem. **06**

Q.3 **(a)** Explain the Pole placement controller strategy in detail. Design a state feedback control law for a given system such that the closed loop system has poles at -1, -2, -3. **12**

$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) What do you mean by Reduced Order Observers? **02**

OR

Q.3 **(a)** A linear autonomous system is described by the state equation **08**

$\dot{x} = Ax$

Where,

$A = \begin{bmatrix} -4k & 4k \\ 2k & -6k \end{bmatrix}$

Find the restrictions on the parameter K to guarantee the stability of the system.

(b) Design a deadbeat state feedback controller for the mixing tank system given in discrete time model as, **06**

$X(k+1) = Fx(k) + Gu(k)$

Where,

$F = \begin{bmatrix} 0.9512 & 0 \\ 0 & 0.9048 \end{bmatrix}$; $G = \begin{bmatrix} 4.88 & 4.88 \\ -0.019 & 0.0095 \end{bmatrix}$

Q.4 **(a)** Find the curve with minimum arc length between the point x(0) =0 and the curve, $\theta(t) = t^2 - 10t + 24$. **06**

(b) Given $\dot{x} = -x + u$, $x(0) = x^0$, $x(2) = x^1$. Find u^* that minimizes $J = \int_0^2 (x^2 + u^2)dt$. **08**

OR

Q.4 **(a)** Derive Euler-Lagrange equation for trajectory $x=x^*(t)$ joining (t_0 , x^0) and (t_1 , x^1) in (t , x) plane such that the functional $J = \int_{t_0}^{t_1} g(x, \dot{x}, t)dt$ is optimal. **06**

(b) A first order system $\dot{x} = -x + u$ is to be controlled to minimize $J = \frac{1}{2}\int_0^1 (x^2 + u^2)dt$. Find the optimal control law. Draw the structure of controller for this system. **08**

Q.5 **(a)** A first order system is described by the differential equation $\dot{x}(t) = 2x(t) + u(t)$. **08**

It is desired to find the control law that minimize the performance index $J = \frac{1}{2}\int_0^{t_1} (3x^2 + \frac{1}{4}u^2)dt$. Also draw the structure of controller for given system.

(b) Write short note on Optimal estimation for linear continuous time system. **06**

OR

Q.5 **(a)** Derive the equations of Optimal control law, Feedback gain algorithm and Optimal performance index for Discrete-time Linear State Regulator problem. **08**

(b) What are the characteristics of Stochastic process? Give the response of linear discrete time system to white noise. **06**

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