Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Regular Examination January / February 2011

Subject code: 710904N

Subject Name: Optimization Techniques

Date: 04 /02 /2011 **Time: 02.30 pm – 05.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Explain and graphically illustrate infeasibility and unboundedness. How can each of these be detected while applying simplex technique? **07**

(b) Use Simplex method to solve the following problem: **07**

Maximise $Z = 5X_1 + 10X_2 + 8X_3$

Subject to $3X_1 + 5X_2 + 2X_3 \leq 60$

$4X_1 + 4X_2 + 4X_3 \leq 72$

$2X_1 + 4X_2 + 5X_3 \leq 100$

$X_1, X_2, X_3 \geq 0$

Q.2 **(a)** One unit of product A contributes Rs 7 and requires 3 units of raw material and 2 labour hours. One unit of product B contributes Rs 5 and requires 1 unit of raw material and 1 labour hour. Availability of raw material is 48 units and there are 40 hours of labour. **07**

1. Formulate as a LPP
2. Write its dual
3. Solve the dual and find the optimal product mix and shadow prices of raw material and labour.

(b) A retired person wants to invest upto an amount of Rs 30,000 in fixed income securities. His broker recommends investing in two bonds. Bond A yielding 7% and Bond B yielding 10%. After some consideration, he decides to invest at most Rs 12,000 in Bond B and at least Rs 6,000 in Bond A .He also wants the amount invested in Bond A to be at least equal to the amount invested in Bond B. What should the broker recommend, if the investor wants to maximize his return on investment? Formulate a LP model for this problem and solve this model graphically. **07**

OR

(b) Maximise $Z = 6X_1 + 5X_2$ **07**

Subject to $4X_1 + X_2 \leq 800$

$2X_1 + 3X_2 \leq 900$

$X_1 \leq 180$

$X_2 \leq 320$

$X_1, X_2 \geq 0$

For the given problem X_1 represents one unit of product alpha, X_2 represents one unit of product omega.

Optimum solution is to produce 150 units of alpha and 200 units of omega product.

Carry out the Sensitivity analysis to determine the range within which objective function coefficient can change without changing the optimum solution.

(1) coefficient of X_1 changes, X_2 fix

(2) Coefficeint of X_1 fix and X_2 changes and

(3) Coeffcient of X_1 and X_2 changes simultaneously.

Q.3 **(a)** Maximise $Z = 200X_1 + 300X_2$ **07**

Subject to $2X_1 + 4X_2 \leq 17$
$3X_1 + 3X_2 \leq 15$
$X_1, X_2 \geq 0$ and integer

Solve the Integer-programming problem using cutting plane algorithm.
Solution to the LP relaxation using simplex method is given in the table below:

Basis		bj	X_1	X_2	S1	S_2
300	X_2	7/2	0	1	1/2	-1/3
200	X_1	3/2	1	0	-1/2	2/3
	Cj		200	300	0	0
Δj	Zj-Cj	1350	0	0	50	100/3

(b) What is integer linear programming? Explain the merits and demerits of rounding off a continuous optimal solution to an LP problem to obtain an integer solution. Also give an example each of a pure and a mixed integer linear programming problem. **07**

OR

Q.3 **(a)** The cost to perform different jobs by different workers is given as follows: **07**

WORKER	JOB			
	1	2	3	4
A	90	18	48	50
B	72	28	85	80
C	53	92	12	78
D	20	70	70	25

Obtain optimal assignment of jobs to workers. Use branch and bound method to solve the given integer programming problem.

(b) Explain the significance of post optimality analysis of a simplex linear programming problem. **07**
How does simplex algorithm indicate that:
1. There is an alternate optimal solution?
2. The problem has unbounded optimal solution?
3. The problem has no feasible solution?

Q.4 **(a)** A company has Rs 6,00,000 to invest, and a total of three investments are available to it. If X_j rupees (in lakhs) are invested in investment j , then a net present value(in lakhs of rupees) of $V_j(X_j)$ is obtained, where $V_j(X_j)$'s are given below: **07**
$V_1(X_1) = 7X_1 + 2$
$V_2(X_2) = 3X_2 + 7$
$V_3(X_3) = 4X_3 + 5$
$(X_1, X_2, X_3 > 0)$ and $X_1(0) = X_2(0) = X_3(0) = 0$
The amount placed in each investment has to be in multiple of Rs one lakh only. How should the company invest Rs 6,00,000 in order to maximize the net present value obtained from the investments? Use Dynamic programming method.

(b) Explain dynamic programming. How is it different from linear programming? Distinguish between deterministic and probabilistic dynamic programming and give some examples where dynamic programming may be used. **07**

OR

Q.4 **(a)** Find the dimensions of a box of largest volume that can be inscribed in a sphere of unit radius. Use direct substitution method. **07**

(b) It has been decided to leave a margin of 30 mm at the top and 20 mm each at the left side, right side and the bottom on the printed page of the book. If the area of the page is specified as 5×10^4 mm^2, determine the dimension of a page that provide the largest printed area. **07**

Q.5 **(a)** Find the dimension of a cylindrical tin (with top and bottom) made of sheet metal to maximize its volume such that the total surface area is equal to $A_0 = 24\ \Pi$.. Use the langrange multiplier method. **07**

Max $f(X_1, X_2) = \Pi X_1^2 X_2$

Subject to $2\ \Pi X_1^2 = 2\ \Pi X_1 X_2 = A_0 = 24\ \Pi$

(b) Find extreme points of the function **07**

$f(X_1, X_2) = X_1^3 + X_2^3 + 2X_1^2 + 4X_2^2 + 6$

OR

Q.5 **(a)** Find maximum of the function $F(X) = 2X_1 + X_2 + 10$ subject to $g(X) = X_1 + 2X_2^2 = 3$ using the langrange multiplier method. Also find the effect of changing the right hand side of the constraint on the optimum value of f. **07**

(b) State the necessary and sufficient conditions for the minimum of a function f(X). Also determine the maximum and minimum values of the function: **07**

$f(X) = 12X^5 - 45X^4 + 40X^3 + 5$

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