**Seat No.: _____** **Enrolment No._____**

# GUJARAT TECHNOLOGICAL UNIVERSITY

## ME Semester –I Examination Feb. - 2012

**Subject code: 710107N** **Date: 16/02/2012**

**Subject Name: Quantum Theory & Algorithm Design**

**Time: 10.30 am – 01.00 pm** **Total Marks: 70**

## Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** **1.** A quantum system is in the state **04**

$$|\Psi\rangle = \frac{3i|0\rangle + 4|1\rangle}{5}$$

(A). Is the state normalized?

(B). Express the state in the $|+\rangle, |-\rangle$ basis

**2.** Explain inner products with suitable example **03**

**(b)** Find out shortest path of spanning tree using Dijkstra's algorithm with vertex node 'a' as the source node for figure 1. **07**

**Q.2** **(a)** **1.** Explain inner product and matrix representation. **03**

**2.** Find MST of graph in figure 2, using Prim's Algorithm. Comment on the running behavior of the algorithm **04**

**(b)** **1.** Perform topological sorting on graph in figure 3. **04**

**2.** Divide and conquer v/s Greedy Approach **03**

**OR**

**(b)** Frequency of the characters 'a' to 'f' is first six prime numbers respectively. Produce the prefix code for all characters using Huffman code algorithm. Adjacency list v/s adjacency matrix representation of graph **07**

**Q.3** **(a)** **1.** Show that the trace of a matrix is equal to the sum of its eigenvalues for **04**

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix},\ B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 4 \\ 1 & 0 & 2 \end{pmatrix}$$

**2.** Using the basis states **03**

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad |-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Construct the projection operators $P_+$ and $P_-$, and show that they satisfy the completeness relation.

**(b)** **1.** Given that $\langle a|b\rangle = \frac{1}{2}$ *and* $\langle c|d\rangle = \frac{3}{4}$, calculate $\langle\Psi|\phi\rangle$, where $|\Psi\rangle = |a\rangle \otimes |c\rangle$ and $|\phi\rangle = |b\rangle \otimes |d\rangle$ **04**

**2.** Calculate the tensor product of $|\Psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, and $|\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$ **03**

**OR**

**Q.3** **(a)** **1.** Find the eigen value end eigen vector for $\pi/8$ gate, which has the matrix representation **04**

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

**2.** Find $X \otimes Y \,|\Psi\rangle$, where **03**

$$|\Psi\rangle = \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$$

**(b)** **1.** A three qubit system is in the place **04**

$$|\Psi\rangle = \left(\frac{\sqrt{2}+i}{\sqrt{20}}\right)|000\rangle + \frac{1}{\sqrt{2}}|001\rangle + \frac{1}{\sqrt{10}}|011\rangle + \frac{i}{2}|111\rangle$$

(A). What is the probability that the system is found in the state $|001\rangle$ if all three qubits are measured.

(B). What is the probability that a measurement on the second qubit only gives 1? What is the postmeasurement state of the system? Show that postmeasurement state is normalized.

**2.** Calculate the matrix representation of $Z \otimes X$, showing that $X \otimes Z - Z \otimes X$. **03**

**Q.4** **(a)** **1.** For the density matrix given by $\rho = \frac{1}{5}\begin{pmatrix} 3 & 1-i \\ 1+i & 2 \end{pmatrix}$ **04**

(A). Is this a mixed state?

Find <X>, <Y> and <Z> for this state.

**2.** Suppose that **03**

$$\rho = \begin{pmatrix} \frac{1}{3} & \frac{i}{3} \\ \frac{-i}{3} & \frac{2}{3} \end{pmatrix}$$

(A). Show that $\rho$ is Hermitian and has positive eigen values that satisfies $0 \leq \lambda_i \leq 1$, and $\sum \lambda_i = 1$

(B). Is this a mixed state?

(C). Find <X> for this state

**(b)** **1.** A two qubit system is in the state **04**

$$|\Phi\rangle = \frac{1}{\sqrt{6}}|01\rangle + \sqrt{\frac{5}{6}}|10\rangle$$

Is the state normalized? An X gate is applied to the second qubit. After this is done, what are the possible measurement results if both qubits are measured, and what are the respective probabilities of each measurement result?

**2.** Draw a binary tree containing keys W, O, R, L, D, such that the post order traversal visits node in R, O, W, L, D and preorder traversal visits node in D, R, L, O, W order. **03**

**OR**

**Q.4** **(a)** Consider an ensemble in which 40% of the systems are known to be prepared in the state **07**

$$|\Psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$$

And 60% of the systems are prepared in the state

$$|\varphi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

(A). Find density operator for each of the states and show they are pure states. If measurements are made on systems in each of these states, what are the probabilities they are found to be in states $|0\rangle$ and state $|1\rangle$ respectively?
(B). Determine density operators for ensemble
(C). Show that $Tr(\rho) = 1$
(D). A measurement of Z is made on a member drawn from the ensemble. What are the probabilities it is found to be in states $|0\rangle$ and state $|1\rangle$ respectively?

**(b)** **1.** A qubit is in the state **04**

$$|\Psi\rangle = \frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle$$

A measurement with respect to Y is made. Given that the eigenvalues of the Y matrix are $\pm 1$, determine the probability that the measurement result is +1 and the probability that the measurement result is -1

**2.** List three problems for which greedy approach always yields optimal solution. **03**

**Q.5** **(a)** **1.** Find Longest Common Subsequence for given strings. **04**

A = XXYZWXYWX
B = YZWXYXY

**2.** Write an algorithm to compute $x^n$ in less than O(n) time **03**

**(b)** Solve Knapsack problem for W = <1, 3, 4, 5, 8, 9>, V = <1, 4, 7, 10, 11, 16> using Greedy Approach. Knapsack capacity is 23 units. Select items in descending order of profit. **07**

**OR**

**Q.5** **(a)** **1.** In which case following sorting techniques are ideal to use: Selection sort, Insertion sort, Merge sort, Quick sort **04**

**2.** Discuss the merits/demerits of Top down dynamic programming and bottom up dynamic programming in reference to chain matrix multiplication. **03**

**(b)** Solve the linear programming for given data: **07**

Minimize -2x1 + 3x2
Subject to
X1 + x2 = 7
X1-2x2 <= 4
X1 >= 0

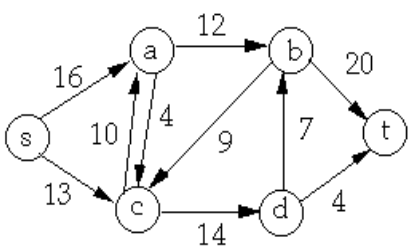


**Figure 1**

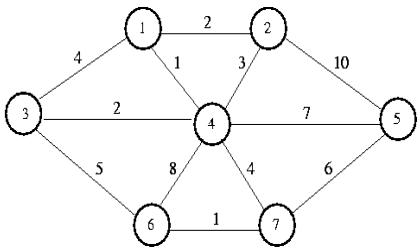


**Figure 2**

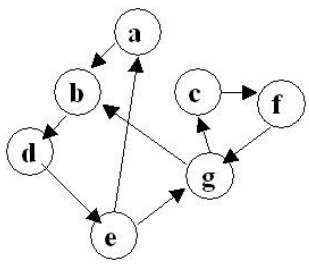


**Figure 3**

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