

GUJARAT TECHNOLOGICAL UNIVERSITY
ME Semester –I Examination Feb. - 2012

Subject code: 710301N

Date: 11/02/2012

Subject Name: Control Engineering

Time: 10.30 am – 01.00 pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Define: 08

- (i) Asymptotically stable in-the-large
- (ii) Asymptotically stable at the origin
- (iii) Total stability
- (iv) Admissible control

(b) Prove that if the system, $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$ is 06
controllable and $b_i (\neq 0)$ is the i th column of B , then there exist a feedback matrix K_i such that the single-input system $\dot{x} = (A+BK_i)x + b_i r_i$ is controllable.

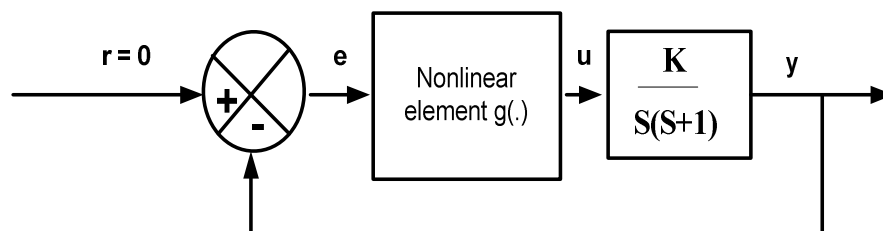
Q.2 (a) Prove that the state model, $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$ is 08
Bounded Input Bounded Output stable if and only if $H(t) = Ce^{At}B$ satisfies, $\int_0^\infty \|H(\tau)\| d\tau = N < \infty$ for all $i = 1, 2, 3, \dots, q$
 $j = 1, 2, \dots, p$

(b) Check the stability of the following system using R-H criteria 06
(i) $3\lambda^4 + 10\lambda^3 + 5\lambda^2 + 5\lambda + 2 = 0$
(ii) $\lambda^6 + \lambda^5 + 3\lambda^4 + 3\lambda^3 + 3\lambda^2 + 2\lambda + 1 = 0$

OR

(b) Derive the equation of performance index for minimum fuel problem 06
and state regulator problem.

Q.3 (a) Consider the nonlinear system shown in figure, where the nonlinear 12
element is given as $u = g(e)$



Find the condition for asymptotic stability using Krasovskii method.

- (b) Draw the structure of Full-order state observer. 02

OR

- Q.3** (a) For the linear system described by the transfer function 09

$$\frac{Y(s)}{U(s)} = \frac{10}{s(s+1)(s+2)}$$

Design a feedback controller with a state feedback so that the eigen values of closed loop system are at -2, -1, $\pm j1$.

- (b) Find the extremals for the following functional: 05

$$J(x) = \int_0^{\pi/4} (x^2 - \dot{x}^2) dt ; x(0) = 0, x(\pi/4) \text{ is free}$$

- Q.4** (a) State and explain Principle of Causality and Principle of Invariant Imbedding for Optimal control system. 06

- (b) Find the optimal control $u^*(t)$ for the system $\dot{x} = u$; $x(0) = 1$ 08

$$\text{Which minimizes } J = \frac{1}{2}x^2(4) + \frac{1}{2} \int_0^4 u^2 dt$$

OR

- Q.4** (a) Derive the fundamental necessary condition for the optimization of fixed end points problem. 05

- (b) Derive the equation of state feedback control law for the continuous time linear state regulator system. 09

- Q.5** (a) Show that the extremal for the functional, 08

$$J(x) = \int_0^{\pi/2} (\dot{x}^2 - x^2) dt$$

Which satisfies the boundary conditions $x(0) = 0$; $x(\pi/2) = 1$ is $x^*(t) = \sin t$.

- (b) Give the response of linear discrete time system to white noise. 06

OR

- Q.5** (a) Derive the equation of the feedback matrix K for time invariant linear state regulator system. 08

- (b) Define stochastic vector process and discrete white noise. Draw the structure of plant with optimal estimator. 06

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